LABOUR-TIME PRICES OF PRODUCTION UNDER ACCUMULATION

*Branko HORVAT**

1. Introduction

Labour theory of prices has been linked almost exclusively with the Marxian analysis of exploitation under capitalism. Various economists have tried to improve on Marx — or refute him — by using his assumptions and applying more advanced mathematics. I am not interested in this century old debate, but shall explore a possible approach to a theory appropiate for a socialist economy. For this purpose I shall make use of the familiar input-output framework. There will be three branches of production:

1. production of machines (M),
2. production of raw materials and intermediate goods (S), and
3. production of consumer goods (C).

In addition, at a particular time, there will be two scarce sources:

1. labour (R) and
2. capital (machines, M).

Our model production will have the following characteristics:

1. The system will be closed, i.e., there will be no foreign trade transactions.

2. Final consumption will consist of personal consumption and investment in fixed capital. This means that complications which follow from state intervention and inventory accumulation will be neglected. In fact it is not too difficult to take also these two factors into account. It can be assumed, for example, that public consumption is financed out of personal incomes and that it is thus included in personal consumption. It can also be assumed that inventory accumulation is proportional to the increase in production. However, the results of the analysis would not be changed by this somewhat more complicated approach.

* The author is Professor of Economics in the Institute of Economic Studies in Belgrade. He wishes to thank dr. Helen Kramer for the translation of the paper into English.

3. The classification of production will be complete, which means that the final branches (M and C) will produce exclusively for final consumption, and the intermediate branch (S) exclusively for intermediate consumption. In reality, of course, the situation is not so clear-cut. For example, the same coal can be used both in the household (final consumption) and the electricity plant (intermediate consumption). It can therefore be taken that the model implies either (a) that the part of production of coal intended for final consumption is included in branch 3 or (b) that 'housing and household management' is included as a separate productive activity in branch 3.

If we define the input per unit of production in the usual way as the technical coefficient

$$b_{rs} = \frac{x_{rs}}{X_s} \tag{1}$$

where r indicates the row and s the column from the input-output table, then balancing the table according to rows gives the well-known expression.

$$BX + Y = X \tag{2}$$

where $B = (b_{rs})$ represents the matrix of the technical coefficients of the intermediate field, Y the column vector of final consumption, and X the column vector of total production. Separate technical coefficients represent the ratios of primary resources and production. These are coefficients of capital, $k_s = M_s/X_s$, and of labour, $\rho_s = R_s/X_s$.

Other symbols and important concepts which will be used in this work are the following:

M = number of machines
S = raw materials and intermediate goods measured in tons
C = number of baskets of consumption goods
Y_s = final consumption of products s
X_s = total output of industry s
A_s = depreciation in industry s
p_M, p_S, p_C = prices of machine-years, intermedite and consumption goods
P_M = price of machine
M_g = number of machine-years
R_g = number of worker-years
n = life-span of a machine
w = real rate of pay, i.e., personal income in baskets of consumer goods per worker annually
$\hat{w} = w\, p_C$ = nominal rate of pay
$\hat{\varkappa} = \varkappa_s P_M/p_s$ = the capital coefficient expressed in value terms

$k = M/R$ = capital-labour ratio or the technical composition of the factors
$\omega = MP_M/R_g = k\,P_M$ = capital-labour ratio in value terms or the organic composition of the factors.

As can be seen, traditional terminology has been taken over, except that the term 'composition of the factors' in the sense of ratio of primary resources, is used instead of Marx's term 'the composition of capital', which is meaningless in this context.

2. Simple Reproduction

In the case of simple reproduction total annual production of machines must be equal to the machines expended in the course of the year. We proceed from the following assumptions:

1. Technology is unchanged and therefore the technical coefficients remain the same.

2. All machines are the same and their life span is n years.

3. In the course of their life span, the capacity of the machines remains constant. After they are expended, the machines have no economic value.

Table 1

	M_g	S_t	C	Y	X
M_g	—	—	—	36	36
S_t	36	24	60	—	120
C	—	—	—	60	60
					Σ
R_g	12	24	30		66
$A = M_g$	12	12	12		36
Z	4	4	4		12

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0{,}2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho' = \begin{bmatrix} \frac{1}{3} & 0.2 & 0.5 \end{bmatrix} \tag{3}$$

$$\varkappa' = \begin{bmatrix} \frac{1}{3} & 0.1 & 0.2 \end{bmatrix}$$

4. Labour is homogeneous and reduced to uniform simple labour.

5. The number of workers remains unchanged in the course of time.

6. Production is at full capacity so that both labour and capital resources are fully employed.

7. Each department produces only one product.

8. There is no working capital, which follows (a) from the previously assumption of non-existence of inventories, and (b) from two implicit assumptions of (i) non-existence of money and (22) payment of personal incomes after completion of production cycle. Since labour force does not increase, the necessary inventories of consumer goods exist and are directly renewed among the population.

It will be useful to illustrate the further course of the analysis with numerical examples. Let the life span of a machine be $n = 3$ years. Let the stock of machines be balanced by age and amount to 36 machines. Accordingly, every year 36/3 = 12 machines must be replaced. The existing stock of machines gives annual service to the amount of 36 machine years (M_g). In annual machine production future services of $12 \cdot n = 36\ M_g$ are also embodied. It may be shown that it is more appropriate to measure depreciation and production in machine years and stock of capital (K) and replacement (Z) by number of machines. Labour services will be measured by worker-years (R_g). Production of raw materials and intermediate goods will be measured in tons, and production of consumer goods by baskets of a standard assortment of consumer articles. Given these conditions, let the transactions and technical coefficients of our production system be as demonstrated in Table 1. It is useful to observe that in the left-hand table $R_g = R$, $M_g = M$ holds for the primary resources: since transactions relate to one year, the quantity of services and the number of resources are numerically the same.

Production being expressed in natural units, direct aggregation is possible only along the rows of the table. Until we determine the weights which will represent prices, aggregation along columns is possible only by means of production functions which tell how much production is obtained from a certain composition of inputs. The structure of the production of our three departments follows from the table:

$$\begin{aligned} &\text{I}\ \ 12\,M_g + 36\,S_t + 12\,R_g \rightarrow 36\,M_g \\ &\text{II}\ \ 12\,M_g + 24\,S_t + 24\,R_g \rightarrow 120\,S_t \\ &\text{III}\ \ 12\,M_g + 60\,S_t + 30\,R_g \rightarrow 60\,C \end{aligned} \tag{4}$$

Now the production system is shown in the (natural) form of Marx's model of reproduction. The first two elements (M_g and S_t) correspond to his constant capital (c), and the third (R_g) to variable capital (v). The system could be even further simplified by showing production of I and II in net form. Then, by subtracting depreciation ($A = 12\,M_g$) in I we could obtain $36 - 12 = 24\,M_g$ of net produot, and by subtracting intermediate inputs in II we could obtain $120 - 24 = 96\ S_t$ of net product. However, we will not carry out that subtraction, for it does not much facilitate the techniques of calculation, and the complete form of the reproduction model makes possible a certain additional insight into the functioning of the system.

The following characteristics of the production system follow from (3) and (4):

1. Annual depreciation in all three departments is equal to the annual production of machine industry,

$$A = X_M \text{ or } (12 + 12 + 12)\ M_g = 36\ M_g$$

2. Annual intermediate consumption is equal to the annual production of intermediate goods department,

$$\sum_s x_{2s} = X_s \text{ or } (36 + 24 + 60)\ S_t = 120\ S_t$$

3. Real personal income amounts to

$$w = \frac{60\ C}{(12 + 24 + 30)\ R_g} = \frac{10}{11}$$

baskets per worker annally.

4. The capital-labor ratio (Marx's technical composition of capital), $k = M/R = M_g/R_g$, in the three departments and for the system as a whole is

$$k_M = \frac{12}{12} = 1,\ k_s = \frac{12}{24} = \frac{1}{2},\ k_c = \frac{12}{30} = \frac{2}{5},\ k = \frac{36}{66} = \frac{6}{11}$$

machines (or machine years) per worker (or worker-year).

If we wish to express the value of production in labour time, say in labour-years, then it is necessary to multiply the inputs and products in (4) by the corresponding prices p_i. In so doing the price of a labour-year must be equated with one, i.e. $p_R = 1$. We obtain a system with three equations and three unknown prices, which is easily solved.

$$\begin{aligned} &\text{I}\ \ 12\,p_M + 36\,p_S + 12\,p_R = 36\,p_M \\ &\text{II}\ \ 12\,p_M + 24\,p_S + 24\,p_R = 120\,p_S \\ &\text{III}\ \ 12\,p_M + 60\,p_S + 30\,p_R = 60\,p_C \end{aligned} \tag{5}$$

From this follow the prices:

$$p_M = \frac{14}{13} R_g,\ p_S = \frac{5}{13} R_g,\ p_C = \frac{14.3}{13} R_g \tag{6}$$

It should be borne in mind that p_M represents the price a machine-year. The price of a machine is n times greater,

$$P_M = 3\, p_M = \frac{42}{13} R_g$$

5. If we add the three equations in (5), we obtain a characteristic identity:

$$(12 + 24 + 30)\, p_R = 60\, p_C \qquad \text{or } R_g = C p_C \tag{7}$$

The labour value of consumer goods is equal to the total expended living labour in the course of the year. From (7) follows that

$$p_C \frac{R_g}{C} = \frac{1}{w} = \frac{14.3}{13} = \frac{11}{10} \tag{8}$$

i.e. the labour-time price of a basket of consumer goods is equal to the reciprocal value of the real rate of pay.

We can now express system (4) in value terms:

$$A_s\, p_M \div x_{2s}\, p_S + R_s\, p_R = X_s\, p_s$$

$$\begin{aligned}
&\text{I} && \frac{168}{13} + \frac{180}{13} + 12 = \frac{504}{13} \\
&\text{II} && \frac{168}{13} + \frac{120}{13} + 24 = \frac{600}{13} \\
&\text{III} && \frac{168}{13} + \frac{300}{13} + 30 = \frac{858}{13} \\
&\sum && \frac{504}{13} + \frac{600}{13} + 66 = \frac{1962}{13}
\end{aligned} \tag{9}$$

The total gross value of production amounts to $X = 1962/13 = 151$ labour-years. Of the total, 66 labour-years represent living labour or newly created value, and the remaining 95 labour-years embodied labour or transferred value. The latter consits of 600/13 labour-years of intermediate goods produced in the current year and $504/13 = 38.8$ labour-years of depreciation, which represents embodied labour from earlier years. In the categories of social accounting, national income is 66 R_g, national product is $66 + 38.8 = 104.8\ R_g$, and gross turnover $104.8 + 46.2 = 151\ R_g$.

6. Finally, we can also determine the organic composition of resources defined as the ratio of past (embodied) and living labour:

$$\omega = \frac{MP_M}{R_g} = k\, P_M$$

It is immediately observed that the organic composition is proportional to the technical composition in which the factor of proportionality is the labour-time price of a machine.

$$\omega_M = 1 \left(\frac{42}{13}\right) = \frac{42}{13}, \quad \omega_S = \frac{1}{2}\left(\frac{42}{13}\right) = \frac{21}{12}$$

$$\omega_C = \frac{2}{5}\left(\frac{42}{13}\right) = \frac{16.8}{13}, \quad \omega = \frac{6}{11}\left(\frac{42}{13}\right) = \frac{22.9}{13}$$

7. The capital coefficient in value terms is

$$\hat{\varkappa}_s = \frac{M_s\, P_M}{X_s\, p_s} = \varkappa_s \frac{P_M}{p_s}, \quad \hat{\varkappa}_M = \frac{1}{3}\,\frac{42/13}{14/13} = 1$$

$$\hat{\varkappa}_S = 0{,}1\, \frac{42/13}{2/13} = \frac{4.2}{5}, \quad \hat{\varkappa}_C = 0.2\, \frac{42/13}{14.3/13} = \frac{8.4}{14.3}$$

Now it is also possible to express the gobal capital coefficient for the whole economy,

$$\hat{\varkappa} = \frac{504.3}{13} \Big/ \frac{1962}{13} = \frac{84}{109}$$

which was not computable earlier when prices were lacking.

Further elaboration of the simple reproduction case as well as of certain kinds of technological progress has been done elsewhere,[1]) and need not be dwelt upon here.

3. Once For All Accumulation

The simplest approach to expanded reproduction lies in consideration of once for all accumulation. In a balanced stationary system there is no possibility of accumulating capital. Accumulation can be achieved either by outside intervention or by restructuring the system. In the first case additional more efficient, machines can be imported as a gift or on credit. (We have already examined the possibilities to increase the efficiency of a machine. On the other hand we do not wish to complicate our model with monetary credit relationships and shall do no more but mention this point). In the latter case production of machines can be increased only at the expense of a reduction of consumer goods production. This represents a change of the population's time preferences. We shall assume that society has decided to achieve the initial accumulation necessary for starting growth by saving on current consumption.

[1]) See B. Horvat (1973).

Let the investment decision consist of the addition of two new machines of n durability to the existing stock of machines. Output of the machinery industry must be increased by $2n\ Mg$. We assume that this is achieved with the least possible structural changes: the structural coefficients of the machinery industry and production of intermediate goods remain the same and department II absorbs the entire change. For the sake of greater perception, the above changes are printed in our numerical example in boldface.

	Mg	St	C	Y	X
Mg	0	0	0	42	42
St	42	24	54	—	120
C	0	0	0	50	50
					Σ
Rg	14	24	28	—	66
A=Mg	14	12	10	—	36

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0.2 & 1.08 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\rho' = [1/3 \quad 0.2 \quad 0.56]$$

$$\varkappa' = [1/3 \quad 0.1 \quad 0.2]$$

$$(I-B)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 10/8 & 10/8 & 10.8/8 \\ 0 & 0 & 1 \end{bmatrix} \quad U_{RX_s} = \rho' (I-B)^{-1} = \left[\frac{7}{12} \quad 0.25 \quad 0.83\right]$$

$$U_{MX_s} = \varkappa' (I-B)^{-1} = \left[\frac{11}{24} \quad 0.125 \quad 0.335\right]$$

It is seen that the only change in the structural coefficients lies in the assumption that because of the disturbance in department III the technical and labor coefficient are somewhat increased. There are more changes in the transactions table. In order to increase production of machines by 1/6, it was necessary to transfer two machines and two workers from the consumption to the machine sector. Because of that there arose a redistribution in the consumption of intermediate goods, and production of baskets of consumer goods fell from 60 to 50 units. Given a constant labor force, the latter means a reduction of real personal income by 1/6.

Because of the unchanged technical and factor coefficients as well as the structure of the technical matrix B, the total coefficients of labor and capital remain the same in the first two departments, $U_{RM} = U^0_{RM}$, $U_{RS} = U^0_{RS}$. Because of a reduction of the efficiency of production, in department III the total coefficients increase. This means that the labor-time prices of the means of production remain unchanged, $p_M = p^0_M$, $p_S = p^0_S$, but the prices of consumer articles increase, $p_C > p^0_C$. Since total living and, due to unchanged depre-

ciation, total embodied labor remain the same, neither the total labor value of production nor the organic composition of the whole system change.

The prices of machine years and intermediate goods are given in (6) and amount to $p_M = 14/13$, $p_S = 5/13$, while p_C can be calculated from the equation for department III (15.48/13 compared to the previous 14.3/13). The value structure of the system is thus:

$$A_s\, p_1 + \quad x_{2s}\, p_2 + R_s\, p_R = X_s\, p_s \tag{10}$$

Ia $14 \times 14/13 + 42 \times 5/13 + 14 \times 1 = 42 \times 14/13 = 588/13$

Ib $12 \times 14/13 + 24 \times 5/13 + 24 \times 1 = 120 \times 5/13 = 600/13$

II $10 \times 14/13 + 54 \times 5/13 + 28 \times 1 = 50 \times 15.48/13 = 774/13$

$36 \times 14/13 + 120 \times 5/13 + 66 \times 1 = 1962/13$

National income (newly created value) amounts as before to 66 labor-years. Gross national product (value added) is equal to the sum of depreciation and national income and amounts, as before, to

$$GNP = Ap_M + R_g p_R = 36 \times 14/13 = 66 = 1362/13 \tag{11}$$

to
or/the total value of final production

$$GNP = M_g p_M + Cp_C = 588/13 + 774/13 = 1362/13. \tag{12}$$

Two consequences follow from the equality of (11) and (12). First of all, accumulation (net investment) appers as the difference between production of department I and total depreciation and as the difference between expended living labor and the labor value of production in department III

$$I \times p_M = (Mg - A)\ p_M = Rgp_R - Cp_C = (42-36)\ 14/13 = 66-774/13 = 84/13 \tag{13}$$

Then, if we express the equality with the aid of the elements which enter into the aggregate,

$$(A_1 + A_2 + A_3)\, p_1 + R_1 + R_2 + R_3 = A_1\, p_1 + x_{21}\, p_2 + R_1 + A_3\, p_1 + x_{23}\, p_2 + R_3,$$

after shortening we obtain

$$A_2 p_1 + R_2 = (x_{21} + x_{23})\, p_2 \tag{14}$$

The added value in the intermediate goods department II is equal to the value of the deliveries of that department to the final departments.

We have just shown, that due to accumulation the volume of living labor is greater than the labor value of the articles of consumption. The difference represents Marx's surplus value, and the ratio between accumulation and expended labor represents the rate of surplus value.

$$\mu = (Rg - Cp_C)/Rg = 1 - Cp_C/Rg = 1 - \frac{774/13}{66} = 1 - 0.9 = 0.1 \qquad (15)$$

The ratio $\hat{w} = Cp_c/Rg$ represents the nominal rate of pay. In a system of simple reproduction $Cp_c = Rg$ and therefore $\hat{w} = 1$. When accumulation occurs, a difference appears —surplus value $m = Rg - Cp_c$ — and the nominal wages fall below one, $\hat{w} < 1$. This means that the worker does not obtain the full ejuivalent of his labor, a difference emerges between the price of labor, (p_R), and the price of labor power as resource (the pay rate), and this difference is exactly equal to the rate of surplus value,

$$\mu = p_R - \hat{w} \qquad (16)$$

The rate of pay being necessarily same in all industries, it follows from (16) that the rate of surplus value will also be same in all departments. This is one of the well known assumptions in Marx's analysis of reproduction.[2])

Since net investment is two machines, the rate of accumulation will be $\pi = 2/36 = 1/18$. It follows from (13) that the rate of accumulation expressed in value terms (surplus value in relation to the value of machines in use) is the same as the rate of accumulation expressed naturally:

$$\pi = \frac{(Mg-A)\, p_M}{Mp_M} = \frac{(Mg-A)\, p_M}{Anp_M} = \frac{Mg-A}{nA} \qquad (17)$$

We can therefore consider the latter as rate of profit. While the rate of surplus value is the same in all departments, the rate of profit changes. There is a simple connection between the two rates:

$$\pi = \frac{Rg}{Mp_M}\,\mu = \frac{\mu}{\omega} \qquad (18)$$

Because $\mu = $ const, the rate of profit is inversely proportional to the organic composition of resources and will decrease with an increased organic composition, and vice versa.

[2]) It is worth observing that our definition of the rate of surplus value differs from Marx's. Using the symbols from *Capital*, Marx defines surplus value as the ratio of surplus value and variable capital, m/v. In our analysis it is defined as the ratio of surplus value and newly added value, m/(v + m). The constancy of one rate implies the constancy of the other. Marx's definition is adapted for measurement of rate of exploitation, ours for measurement of distribution of income between personal income and accumulation.

4. Transformation of Labor time Prices into Prices of Production

If the economy consisted of only three industries, the analysis would thus be basically finished with the above considerations. The society might decide to devide the working day into a part devoted to production of articles for consumption and a part for accumulation, and the structural coefficients of the system would determine the production and allocation of the resources in the departments. But if the economy consists of many industries with partially substitutable resources and products the production can be structured in the most varied ways, It is therefore necessary to determine the criterion of structuring.

In that respect Marx's problem was relatively Simple:[3]) in a capitalist system, surplus value must be distributed so that capitals participate proportionaly in total mass of surplus values, or in other words so that the rate of profit is everywhere the same. This is the well known problem of transforming values into prices, which is very simply solved in our system.

If the durability of all the machines is the same depreciation amounts to $A_s = 1/n\; M_s$ everywhere; consequently, profit can be expressed as

$\pi' M_s\, p_M = \pi' A_s\, p_M = \pi'/n\; M_s\, p_M$ and the rate of profit is equal to

$$\pi = \pi'/n \qquad (19)$$

where π' is the equivalent rate of profit calculated for the sake of simpler calculation, on the basis of depreciation. Instead of the price of labor, $p_R = 1$, let us take the price of labor pover, $\hat{w} \gtreqless 1$, which represents the nominal wage. System (10), expressed in symbols, will now look as follows:

$$\begin{aligned} A_1\, p_1\, (1+\pi') + x_{21}\, p_2 + R_1\, \hat{w} &= X_1\, p_1 \\ A_2\, p_1\, (1+\pi') + x_{22}\, p_2 + R_2\, \hat{w} &= X_2\, p_2 \\ A_3\, p_1\, (1+\pi') + x_{13}\, p_2 + R_3\, \hat{w} &= X_3\, p_3 \end{aligned} \qquad (20)$$

The natural magnitudes, given by the structural coefficients and the accumulation decision, remain constant. However, in addition to three product prices, the two prices of the primary resources, $\hat{w}$ and π', will now also appear as unknowns. In order for the system to be determinate, two more equations must be added. These two equations must regulate the relation between accumulation and department I and between consumption and department III. Since the capitalists spend part of their profits for personal consumption, and the workers, in general, spend their wages, $\pi M p_M + A p_1 > X_1 p_1$, $\hat{w}\, R < X_3 p_3$ must

[3]) Nevertheless, because of insufficient mathematical training, Marx did not succeed in solving it correctly. See Sweezy (1949), ch. VII.

hold. It is obvious that all prices change depending on what we assume for these two relationships. Since our concern is not a capitalist but a socialist system with equal rates of profit, in an economy without money and without inventory accumulation, it obviously must hold true that gross profit (net profit plus depreciation) is equal to the value of machines, and personal incomes to the value of articles of consumption produced.

$$\begin{aligned} \pi M p_M + A p_1 &= X_1\, p_1 \\ \hat{w} R g &= X_3\, p_3 \end{aligned} \qquad (21)$$

We insert (21) into (20), taking into account that $\pi M p_M = \pi' A p_1$ and therefore

$$\pi M p_M + A p_1 = A p_1\, (1 + \pi') = X_1 p_1 \qquad (22)$$

The insertion of (22) and (20) gives this system of equations:

$$\begin{aligned} A_1\, p_1\, (1 + \pi') + x_{21}\, p_2 + R_1\, \hat{w} &= A p_1\, (1 + \pi') \\ A_2\, p_1\, (1 + \pi') + x_{22}\, p_2 + R_2\, \hat{w} &= X_2\, p_2 \\ A_3\, p_1\, (1 + \pi') + x_{32}\, p_2 + R_3\, \hat{w} &= \hat{w} R \end{aligned} \qquad (23)$$

We have obtained a system which formally resembles simple reproduction with the difference that the prices of a machine year are increased, $\xi = p_1\, (1 + \pi') > p_1$, and prices of a labor-year are reduced, $\hat{w} < p_R = 1$. Further analysis shows that substitution of (21) transformed (20) into a system of homogeneous equations.

$$\begin{aligned} (A_1 - A)\, \xi + x_{21}\, p_2 + R_1\, \hat{w} &= 0 \\ A_2\, \xi + (x_{22} - X_2)\, p_2 + R_2\, \hat{w} &= 0 \\ A_3\, \xi + x_{32}\, p_2 + (R_3 - R)\, \hat{w} &= 0 \end{aligned} \qquad (24)$$

which means that the ratio of all prices is determined, but not the level. The latter will be determined if we fix one of the prices.

System (10) can also be made homogeneous. From (13) it follows that

$$X_3\, p_3 = R p_R - (X_1 - A)\, p_1$$

If we set $p_R = 1$ we can obtain the labor-time prices (p_s^*). When the above is inserted into (10) we obtain:

$$\begin{aligned} (A_1 - X_1)\, p_1^* + x_{21}\, p_2^* &= -\, R_1 \\ A_2\, p_1^* + (x_{22} - X_2)\, p_2^* &= -\, R_2 \\ (A_3 + X_1 - A)\, p_1^* + x_{23}\, p_2^* &= R - R_3 \end{aligned} \qquad (25)$$

System (24) can be devided by $\hat{w}$ and (25) can be used to obtain the prices of production as a function of labor-time prices

$$\xi = \hat{w}\, p_1^*\, \frac{(A_3 + X_1 - A)\, x_{21} + (X_1 - A_1)\, x_{23}}{A_3\, x_{21} + (A - A_1)\, x_{23}} = \hat{w}\, p_1^* \left(1 + \frac{(X_1 - A)\, (x_{21} + x_{23})}{A_3\, x_{21} + (A - A_1) x_{23}}\right) \qquad (26)$$

$$p_2 = \hat{w}\, \left(p_2^* + p_1^*\, \frac{(X_1 - A)\, A_2}{x_{23}\, A_2 + (X_2 - x_{22})\, A_3} \right)$$

It follows from (26) that:

$$\xi / \hat{w} > p_1^*\,,\quad p_2 / \hat{w} > p_2^*$$

Therefore, if we express the prices of production through nominal rate of pay, they will be higher than labor-time prices. It is necessary, however, to have in view that ξ is some sort of gross price, and that the net price of a machine is

$$p_1 = \xi\, (1 + \pi')^{-1}$$

We express the fraction in the brackets in (26) as:

$$\frac{(X_1 - A)\, (x_{21} + x_{23})}{A_3 x_{21} + (A - A_1) x_{23}} = \frac{(X_1 - A)\, (x_{21} + x_{23})}{A_3\, x_{21} + (A_2 + A_3)\, x_{23}} = \frac{X_1 - A}{A_3 + \dfrac{A_2\, x_{23}}{x_{21} + x_{23}}} >$$

$$> \frac{x_1 - A}{A_3 + A_2} > \frac{X_1 - A}{A} = \pi'$$

so that now we can write

$$p_1 / \hat{w} = \frac{p_1^*\, (1 + a\pi')}{1 + \pi'} > p_1^*\,,\quad a > 1$$

In this way we have determined that the reduction of the nominal value of production by the rates of pay will not, as is generally thought, give the labor content of production but more than that; we will obtain the inflated values for the labor time contained in production.

From (26) it follows that the relative prices in the two systems are different. But if it is taken into consideration that according to (26) p_1 and p_2 are proportional to p_1^* and p_1^*, and the latter prices are correct measures of labor content, the changes in labor content will be observed in the changes in prices of production in the same direction.

It still remains to determine p_3. Because of the special function of the articles of consumption in the production system, we shall determine their prices in a somewhat different way. Since the goal of a socialist production system is the icrease of production of articles of consumption, and consumer valuations determine the structure of production, the theoretically most interesting case is when labor-time (p_3^*) and money (p_3) price of consumer goods are identical ($p_3 = p_3^*$). For that to be attained the following has to hold:

$$X_3 \, p_3^* = X_3 \, p_3 = \hat{w} R g \tag{27}$$

$$\therefore \; \hat{w} = \frac{X_3}{Rg} \, p_3^* = w \, p_3^* \tag{28}$$

It follows that the condition is satisfied when the money rate of pay is equal to the natural personal income valued in working time. This shows that $p_3/\hat{w} > p_3^*$ holds with respect to these prices as well. It is of interest to observe that in simple reproduction the labor-time price of consumer goods is equal to the reciprocal value of the real rate of pay, $p_3^* = 1/w$, and therefore $w = p_R = 1$. In the case of expanded reproduction more labor is expended than contained in articles of consumption, and therefore

$$\frac{Rg - X_3 \, p_3^*}{Rg} = 1 - w p_3^* = \mu$$

whence it follows that labor-time price is lower than the reciprocal of the real wage for the proportion of the rate of surplus value to the real wage,

$$p_3^* = \frac{1 - \mu}{w} \tag{29}$$

Inserting (29) into (28) gives

$$\hat{w} = 1 - \mu$$

which means that the nominal rate of pay must be equal to the »necessary labor time«, i.e. to the proportion of the working year sufficient for the production of articles of consumption.

System (24) is devided by $\hat{w}$ so that the remaining two money prices can be calculated:

$$\xi = \hat{w} \frac{x_{21} \, (R - R_3) + x_{23} \, R_1}{x_{21} \, A_3 + x_{23} \, (A - A_1)} = \hat{w} \, \alpha$$

$$p_2 = \hat{w} \left[A_2 \, (R - R_3 + A_3 \, R_2 \right] / \left[x_{23} \, A_2 + A_3 \, (X_2 - x_{22}) \right] = \hat{w} \, \beta \tag{30}$$

where the coefficients α and β are determined by the parameters of the system.

Both prices rise when R_1 and R_2 increase, which we would have expected. In the expression for ξ the weights are intermediate commodity deliveries, and for p_2 depreciated machines. Because of that

ξ falls when A_2 and A_3 increase, and p_2 falls when x_{21} and x_{23} increase. If $\hat{w}$ is determined so that the labor-time and money price of a consumer commodity are the same, »the price of production« for our system of expanded reproduction will be

$$\xi = \hat{w} \, \alpha = w \, p_3^* \, \alpha \, , \; p_2 = \hat{w} \beta = w p_3^* \, \beta$$

$$p_3 = p_3^* = \hat{w} / w \tag{31}$$

Since the relative prices in both systems are fixed, and p_3^* is the common element, with the aid of p_3^* prices of production can be expressed as functions of labor-time prices. Money prices are directly proportional to the nominal rate of pay as was expected. The money prices of machines and intermediate goods differ from labor-time prices. In addition, ξ represents the »gross« price of a machine year. The net price is obtained by discounting by the rate π':

$$p_1 = \frac{\xi}{1 + \pi'} = \frac{w \, p_3^* \, \alpha}{1 + \pi'} \tag{32}$$

from (22) it follows that

$$\pi' = \frac{X_1}{A} - 1$$

which means that the discount rate π' in fact represents the rise of machine production above depreciation needs.

Let us further consider what will happen with the organic composition of factors and the capital coefficients in value terms. We define the organic composition as

$$\omega = \frac{M p_M}{Rg \, \hat{w}} = k \, \frac{p_M}{\hat{w}} = k \, \frac{n p_1}{\hat{w}}$$

and insert the value for p_1:

$$\omega = k \, \frac{\dfrac{n \, \xi}{1 + \pi'}}{\hat{w}} = k \, \frac{n \, \hat{w} \, \alpha}{\hat{w} \, (1 + \pi')} = k \, \frac{n \, \alpha}{1 + \pi'} \tag{33}$$

The organic composition is invariant to the nominal rate of pay and is determined exclusively by the structure of the system. However, since the relation of prices changed, the organic composition exprĕssed production prices differs from that expressed in labor-time prices.

The capital coefficient expressed in value terms,

$$\hat{\varkappa}_s = \frac{M_s \, p_M}{X_s \, p_s} = \varkappa_s \, \frac{p_M}{p_s} = \varkappa_s \, \frac{n p_1}{p_s} \tag{34}$$

in department I is independent of prices

$$\hat{\varkappa}_1 = n\varkappa_1 = \hat{\varkappa}_1^{*}$$

and is therefore the same as in the system of labor-time prices. In the other two departments the capital coefficient in value terms changes

$$\hat{\varkappa}_2 = \varkappa_2 \frac{\dfrac{n\,\xi}{1+\pi'}}{\hat{w}\,\beta} = \varkappa_2 \frac{n\,\hat{w}\,\alpha}{\hat{w}\,\beta\,(1+\pi')} = \varkappa_2 \frac{n\,\alpha}{\beta\,(1+\pi')}$$

$$\varkappa_3 = \varkappa_3 \frac{n\,\xi}{1+\pi'} \Big/ p_3^{*} = \varkappa_3\, n\hat{w}\alpha \Big/ \hat{w}/w\,(1+\pi') = \varkappa_3\, nw\alpha / (1+\pi')$$

but is also invariant with respect to the nominal rate of pay.

5. Conclusion

We can now summarize. An identity of labor-time prices and price of production is readily succeptible in simple reproduction. In expanded reproduction it can be attained only when growth is conditioned exclusively by technological progress or when profit rates are inversely proportional to the organic composition of factors. If profit rates are proportional to the resources engaged but the organic composition is different in individual departments, price relation schange. Because of that the organic composition and the capital coefficients in value terms will change depending on which price system is used. However, both groups of value indicators remain invariant in relation to the level of the nominal rate of pay. In addition, money prices will change with a change of labor content in the same direction as labor-time prices.

Since the production system expressed in value terms is homogeneous, it is always possible to fix one of the unknowns. Accordingly, it is possible to equate the labor-time and production price for any department and then calculate the other two prices. It is most natural to equate the prices of consumer goods. This implies a nominal rate of pay which corresponds to the labor time necessary to produce articles of consumption and takes care of the equilibrium of the system, in which the wages fund is in its entirety spent on purchasing articles of consumption, and the production of articles for consumption is entirely financed from the wages fund.

We can conclude that the value of products will reflect the labor embodied in those products only in the case when prices are formed on the basis of equal rates of surplus value. Insofar as prices are formed on the basis of some other principle, let us say on the basis of equal rates of profit, only some prices can reflect the labor content of products though all prices will correctly reflect the direction of

change in labor content. In the case of prices of production, prices of fixed capital are obtained by discounting the gross rental of fixed capital. All prices valued in nominal pay rates will be higher than labor-time prices.

In a socialist economy the criterion of distribution of surplus value is not one of ownership but of allocation. That is, prices should be set so that they make possible an optimal allocation of resources, i.e. such use of embodied and living labor that maximally satisfies the needs of the given society. In this connection it is not at all a priori clear whether it is necessary to equalize rates of surplus value or profit rates or do something else. The above summary gives certain indications of the direction in which to move. However, a solution of the problem is not at all easy and demands very serious investigation.

In addition, a definitive solution requires the abandonment of such limitations as »only three branches of production«, absence of inventories, equal and unchanging durability of all machines, absence of non-reproducible material factors, fixed coefficients, »once for all« character of technical progress or accumulation, etc.[4]) This work represents just the first step in the direction of an eventual future reconstruction of economic theory on the basis of labor.

(Rad primljen avgusta 1974.)

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[4]) Some of the items given in the reference list can be consulted for further study. In the respectively quoted articles Johansen (1963) includes consumer preferences in the analysis, Konüs (1964) shows that consumer utility is maximized if the marginal costs of labor (and therefore also the prices of consumer goods) are proportional to the average costs of labor, and Stiglitz (1968) points out that technology which minimizes labor input will under certain conditions be optimal. Informative and at the same time entertaining is Samuelson's survey of earlier solutions along with his own supplementation (1970, 1971): trying to underrate Marx as much as possible, Samuelson shows his lack of understanding of everything except mathematics (the 1970 work therefore entirely correctly published by the American Academy of Sciences in the section for applied mathematics). However Samuelson makes a positive contribution in the joint article with Weizsäcker (1971) in which the consequences of »dating« labor, which leads to a rate of profit equal to the growth rate of the system, are examined.

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ТРУДОВЫЕ ЦЕНЫ ПРОИЗВОДСТВА ПРИ УСЛОВИЯХ НАКОПЛЕНИЯ

Бранко ХОРВАТ

Резюме

Рассматривается вопрос „трудовых цен производства" на основе производственной модели, состоящей из трех отраслей: производства машин, производства материалов и сырья и производства предметов потребления. Автор добился следующих результатов.

Идентичность „трудовых" цен и цен производства возможна лишь при простом воспроизводстве, в то время как при расширенном только в таком случае, если рост обусловлен искючительно технологическим прогрессом, а также если нормы прибыли обратно пропорциональны органическому составу факторов. Поскольку нормы прибыли являются пропорциональными ассигнованным средствам, и органический состав отдельных отраслей разнится, соотношения цен изменяются. Вследствие этого, органический состав и стоимостный капитальный коэффициент изменяются в зависимости од того, какая система цен используется. Однако, обе группы индикаторов стоимости остаются неизменными по отношению к номинальной заработной плате. К тому же, по мере изменения трудового содержания, денежные цены изменяются в том же направлении как и „трудовые".

Поскольку производственная система в стоимостном выражении гомогенна, всегда можно определить одну из неизвестных величин. Следовательно, возможно приравнить трудовую цену к цене производства для любой отрасли и потом вычислить остальные две цены. Естественнее всего было бы приравнить цены предметов потребления. Это подразумевает номинальную заработную плату, соответствующую рабочему времени, необходимому для производства предметов потребления. Это заодно является и условием равновесия для системы в которой фонд заработной платы полностью расходуется на покупку предметов потребления, а производство предметов потребления полностью финансируется фондом заработной платы.

Можно сделать вывод, что стоимость продуктов производства выражает труд овеществленный в этих продуктах производства лишь в таком случае, когда цены образуются при равной норме прибавочной стоимости. Поскольку цены образуются на основе какого-нибудь другого принципа, например, на основе равных норм прибыли, только некоторые цены могут отражать трудовое содержание, хотя на всех ценах точно будет сказываться направление изменений в трудовом содержании. В случае цен производства, цены основных средств получаются дисконтированием бруто ренты основных фондов. Все цены, полученные на основании номинальных заработных платах будут больше трудовых цен.

В социалистическом народном хозяйстве критерий распределения прибавочной стоимости является не собственическим, а критерием развития размещения ресурсов. Собственно, ценообразование должно быть таким, чтобы обеспечить оптимальное размещение ресурсов, т.е. такое использование овеществленного и живого труда, чтобы максимально удовлетворить потребности данного общества. В этой связи априорно отнюдь не ясно, надо ли приравнивать нормы прибавочной стоимости к нормам прибыли или сделать что-то иное. Настоящая статья дает некоторые индикации в каком направлении надо идти. Однако, решение проблемы далеко не просто и требует весьма серьезного исследования.