POTENTIAL G.N.P. ESTIMATION

The main aim of this article is a verification of one method for potential G.N.P. estimation, i.e. estimation of the degree of resource utilisation. This latter coefficient gives a clear notion of the magnitude of an economy's reserves and, in this way, aids the choice of an economic policy. The U.S. economy's data are used for illustrative purposes.

§ 1. *A Survey of the previous theoretical and empirical results*

Potential G.N.P. estimation is an instrument for a fairly adequate description of the economy's production loci under conditions of the maximal rate of resource utilisation. The concept of the rate of resources utilisation is a very subtle and perhaps a vague one because a 100% resource load is quite impossible both technically and economically.

For this reason two groups of methods in principle can be used with different degrees of success for potential G.N.P. estimation.

The first group is based — in an ideal situation — on the estimation of the distance between 100% and actual rates of resource utilisation (1—2). But in this case it is necessary to assume the existence of an optimal plan (in some sense). Therefore, the second subgroup of these methods takes a step toward reality and overcoming the difficulties in the area of measurement: the rate of resource utilisation is estimated as the difference between its values in the second best and in the actual economy (3—4).

The second group is also heterogeneous. One of the subgroups is based on the »ceiling« concept — in an engineering or economic sense — of capacity in different industries with subsequent aggregation of capaicties in a single number. For this purpose a production function or input-output method can be used (5—9).

The second subgroup includes the following varieties:

1) indirect estimation of potential G.N.P. using a polynomial connecting the potential G.N.P. with the unemployment rate (10—13), quite independent from the general production function,

2) potential G.N.P. estimation with the help of a trend for the peak-years (Wharton School and Federal Reserve Board methods) (14—15),

3) potential G.N.P. estimation in the general context of a macroeconomic production function.

In this case there are two main varieties: a) an estimation of the parameters in an exponential polynomial (with the unemployment rate as an argument) which make a multiplicative part of a macroeconomic production function (16—17); b) an iterative simultaneous estimation of the potential G.N.P. and the production function's parameters using the residual in the production function and as a first approximation the Wharton School estimate of the potential G.N.P. (18).

All methods in the first group depend to a high degree upon the quality of the expert's evaluations and on the exactness of their aggregation. In the second group the FRB and Wharton School methods assume the availability of fairly exact information concerning peak-years capacities in different industries and make the results of the entire estimation procedure dependent upon a trend based on a very small sample. Finally, estimation of the potential G.N.P. in the context of production function estimation in any case depends in a crucial way upon the general economic assumptions in the theory of production functions and the concrete form of its specification — conditions much broader than the problem of potential G.N.P. evaluation.

As a net result, we have at our disposal only statistical methods connecting potential G.N.P. with the actual rate of unemployment and the rate of full employment (97%). But here we also must choose between the Black-Russell and Phelps procedures (13, 10).

Black and Russell's method first estimates the potential labor-force through the proportion of employed persons (full employment is equal to 96%), then evaluates long-run demand for the labor force using a two-factor Cobb-Douglas function with an adaptive mechanism, the introduction of man-hours data, and finally an application of the resulting first order stochastic difference equation to the estimation of the potential G.N.P.

The main advantages of this procedure consist in the presence of the adaptive elements and in simultaneous estimation both of the potential labor force and potential G.N.P. The method has also two main weak points:

1) the application of a in some degree conditional procedure for the evaluation of the potential labor force (for this it is necessary to estimate male employment in the 25—54 age group and the employment of all other persons aged 14 years and more),

2) all results in a crucial way depend upon the properties of the two-factor Cobb-Douglas production function. But its shortcomings are well known and adequacy as a description of the U.S. economy is questionable.

It seems, therefore, that at present time the use of Phelps method is most expedient because in essence it does not depend upon dubious economic assumptions.

All empirical results concerning the potential G.N.P. are based on the concept of fullemployment of labor (96—97%) with a constant traditional number of shifts, but not on the full use of working time concept (in some respects (13) is an exception because there the number of man-hours is used for the measurement).

In other words, all contemporary western estimates are based on a notion about purely economic boundaries of the potential G.N.P.'s size: the latter is defined under 96—97% rate of employment and in conditions of constant number of shifts, traditionally fairly near to 1.

On the other side is situated the notion of technical capacity which in the thirties was applied both to a single branch and to the industry as a whole (19—21). This concept implies that capacity is defined either conditionally using the full number of working places (then we did not take into account the technical capacity and the number of shifts) or on the basis of a 20-hour working day and 310 working days in the year. This latter definition corresponds fairly well to the technical capacity of an economy in the emergency case.

An intermediate position is occupied by the notion of a capacity and a potential G.N.P. based on the estimation of usable working time with the constant traditional number of shifts. It seems that this concept is most adequate for G.N.P. estimation when the economy works under normal conditions but capacity is defined in some interval between its economical and technical boundaries.

More precisely, the size of the potential G.N.P. in this intermediate case is defined by the following:

1) the number of shifts must be constant and fairly near to its traditional one-shift level,

2) as an instrument for measurement the number of man-hours must be taken — not the quantity of employed or the working places,

3) all forms of part-time employment must be taken into account: the mean time of unemployment and parttime employment during the year of the persons included in the data,

4) in principle all losses of working time during each day can also be taken into account.

But the corresponding data for the U.S. economy were not at the authors disposal.

Thus, under the conception used later the size of potential G.N.P.-due to the relatively low constant number of shifts — will be essentially larger than under the evaluation with the help of the unemployment rate, but much less than the technically feasible »ceiling« of production possibilities. In this case 3% loss of working time can be considered as inevitable for technical and production reasons.

Now we turn immediately to description of the model to a survey of the results.

§ 2. Phelps model and its application to the estimation of potential G.N.P. and rates of resource utilisation

Later in the paper we shall use the following notation: Y_t, Y_t^* — actual and potential G.N.P. ψ_t, ψ_{1_t}, ψ_{2_t}, ψ_{3_t} general rate of resource utilisation, rates of utilisation for labor force, capital and land respectively.

u_t, u_{1_t} loss of working time in percent, unemployment rate.

α_{1t}, α_{2t} α_{3t} — distribution parameters of the national income. ($\alpha_{1t} + \alpha_{2t} + \alpha_{3_t} = 1$)

According to the definition

$$\Psi_t = \frac{Y_t}{Y_t^*} = exp\ (a_1 + b_1 u_t + e_1 u^2_t), \tag{2.1}$$

where a_1 must be choosen so that $Y_t = Y_t^*$ if $u_t = 3.00$, i.e. under the assumption of 97% employment rate of the labor force.

Using now the representation:

$$Y_t^* = Y_t : exp\ (a_o + b_o t + c_o t^2 + d_o t^3 + f_o t^4) \tag{2.2}$$

we get the equation relative to $(a_o + a_1)$, b_1, c_1:

$$ln\, Y_t = a_o + a_1 + b_o t + c_o t^2 + d_o t^3 + f_o t^4 + b_1 u_t + c_1 u^2_t \tag{2.3}$$

with the border condition for the estimation of a_1:

$$exp\ (a_1 + 3\, b_1 + 9\, c_1) = 1 \tag{2.4}$$

(2.1) — (2.4) are sufficient for the potential G.N.P. estimation for time intervals without structural breaks, i.e. on the basis of relatively short time series. In the presence of structural breaks (for example, the years 1917—19, 1942—1945 for the U.S. economy) we begin to feel which pecularities of the adopted conception of the G.N.P., definition manifest themselves in the fact that in a war economy or at times of higly overheated economic activity both the general rate of resorce utilisation and the rate of capital load can be greater than 1 (the upper limit is something like 1,05). This is a sign that for such periods we need some change in the direction of the »technical ceiling« conception of the potential G.N.P. because in similar situations the assumption of the constant traditional number of shifts is unacceptable. In practice this change for such years was taken into account in the following way: both the general rate of resource utilisation and the rate of capital load were equated to 1 and the corresponding values of the potential G.N.P. were reestimated.

Thus we come to the estimation problem of the resource utilisation rates. With respect to the labor force nothing can be done because unemployment data (of either sort) are considered as exogeneous. As far as capital and land are concerned, a two-step iterative procedure based on the successive application of the weighted average (with resources' shares in national income as weights) can be applied.

Because the general rate of resource utilisation is already known as a result of potential G.N.P. estimation, and the distribution parameters are exogeneous data (to be precise, only α_{1_t} and a_{3_t} are exogeneous and $\alpha_{2t} = 1 - \alpha_{1t} - \alpha_{3t}$), we can estimate in the first step the combined rate of capital and land utilisation:

$$\psi_{2+3,t} = (\psi_t - \alpha_1\, \psi_{1_t}) : (\alpha_{2_t} + \alpha_{3_t}) \tag{2.5}$$

It is impossible to suggest any unified — to say nothing about mechanical — procedure of estimation for different categories of land in real use. At the present time we can rely only on very crude expert evaluations. For example, in the case of agricultural land the rate of utilisation can be very approximately equated to the agricultural employment rate, and in mining to the general utilisation rate of capital and land in the economy as a whole; i.e. to $\psi_{2+3,t}$ from (2.5), and so on. In short, ψ_{3_t} at the present time can be represented only as a weighted avorage of different expert data and must be introduced as exogeneous information.

Then the rate of capital utilisation can be found in the second step using (2.5):

$$\Psi_{2t} = (\Psi_{2+3,t} - \alpha_{3t}\ \Psi_{3t}) : \alpha_{2t} \tag{2.6}$$

A computer program for the whole procedure (2.1) — (2.5) is given in appendix 1.

Let us now turn directly to the statistical part — data generation and a description of the results.

Crucial for the whole estimation are the data on the share of lost working time. These losses have a two-fold origin: full unemployment and part-time employment.

The data on the unemployment rate for the period 1900—1947 include all persons aged 14 years or more, and for the period 1948—1966, 16 years or more. The data are seasonally unadjusted. The sources for the years 1900—1966 are (22—23); for the years 1897—1899, (24). In the latter case only industry, construction and transport are represented. The figure for the year 1896 is our estimate.

But the figures on the unemployment rate of persons are not equal to the percentages of the working time lost by these persons due to the sharp fluctuations in unemployment duration in the years 1896—1966. Only for the years 1947—1966 is there direct information on the duration of unemployment (23). In order to obtain approximate estimates for the remaining years we calculated the average change in unemployment duration relative to a 1% change in unemployment. It happens that the average for the years 1949, 1951, 1954, 198, 1961, 1964—1966 is equal to 1,57 weeks) 1% change of unemployment. The application of this coefficient produced plussibla results for the periods 1896—1920 and 1929—1946, while the technological unemployment of the twenties was the reason for minor changes in the values of this coefficient for the years 1921—1928.

Thus we calculated the equivalent of the unemployment rate of persons expressed as a percentage of the working time lost due to full unemployment.

The percentage of the working time lost due to part-time employment was derived in the following way. For the period 1929—1966 there were direct data (25—26) concerning the number of employees and its full-time equivalent. The ratio of the latter to the former gives an approximate idea on the percentage of the working time lost due to part-time employment. The word »approximate« is justified by the fact of limited coverage (only employees). For the years 1914 and 1920—1928 the figures (27, S. 114) are, perhaps, most reliable. Although the coverage is limited only by the trade unions members the data give a direct estimate of working time lost due to part-time employment. For the remaining years the figures (24) on the percentage of part-time employees in industry, construction and transport were considered as an equivalent of the working time lost due to part-time employment. Although the control data (28, p. 257) on the rate of working time lost in the private sector in the years 1899, 1909, 1919 were comparable with ours neither in coverage nor method (they are census data), they gave a rough idea of the plausibility of the results.

The data derived in the way described above are given in appendix 2.

Further, as is seen from (2.5, the estimation of the capital utilisation rate implies the availability of data distribution of national income (in current prices) among the factors of production. More precisely, it is sufficient to estimate only α_{1_t}. The process decomposes in two phases the evaluation of national income in current prices and the estimation of the income imputed to the labor force. A more detailed description together with the results is given in appendix 3.

Now we turn directly to the results. The estimation was made in the following versions: 1) the main version based on the data on the percentage of working time lost for the years 1896—1966; 2) the same version with subdivision into the periods 1896—1914, 1914—1929, 1929—1945, 1945—1966; 3) compared with western estimates version based on the data on the employment rate of persons during 1896—1966, 4) the same version with subdivision into the periods.

In the main version based on the data for the period 1896—1966 the equation (2.3) has the form:

$$ln\, Y_t = 4{,}247 + 0{,}0494 + 0{,}0327\, t + 0{,}000139\, t^2 - 0{,}0000068\, t^3_t + 0{,}000000067\, t^4 - 0{,}0163\, u_t - 0{,}000054\, u^2_t$$

$$\sigma = 0{,}0438, \quad \sigma^* = 0{,}0384, \quad \nu = 0{,}0304, \quad t^*_{a_0+a_1} = 104{,}9:$$

$$t^*_{b_0} = 5{,}24, \quad t^*_{c_0} = 0{,}390, \quad t^*_{d_0} = -0{,}920, \quad t^*_{f_0} = 1{,}33$$

$$t^*_{b_1} = -4{,}73, \quad t^*_{c_1} = -0{,}503 \tag{2.7}$$

where (as in the following) t* is Student t-statistic, $\sigma^* = 1{,}253\,|\eta|$, $|\eta|$ is the moduls of the deviations (divided on $T - {}^1/_2$), the ratio σ/σ^* is used as a criterion for the approximate normality of the distribution, and ν is the coefficient of variation.

The equations for separate periods, except the years 1896—1914, 1946—1966, produced insignificant t*-statistics. In the equation for the period 1946—1966 the estimators were strong biased upward — probably under the influence on the higher degrees in the polynomial. In any case, it was impossible to use this equation in order to improve the results (2.7).

In the case with the data on the unemployment rate during 1896—1966 the equation (2.3) has the form:

$$
\begin{aligned}
ln\, Y_t &= 4{,}1925 + 0{,}0515 + 0{,}0376\, t - 0{,}000099\, t^2 + \\
&+ 0{,}00000003\, t^4 - 0{,}000002\, t^3 - 0{,}0166\, u_t - 0{,}00018\, u^2_t,
\end{aligned}
$$

$$\sigma = 0{,}0344, \quad \sigma^* = 0{,}0301, \quad \nu = 0{,}0065, \quad t^*_{a_0+a_1} = 152{,}2,$$

$$t^*_{b_0} = 8{,}12, \quad t^*_{c_0} = 0{,}373 \quad t^*_{d_0} = 0{,}377, \quad t^*_{f_0} = 1{,}46,$$

$$t^*_{b_1} = 5{,}67 \quad t^*_{c_1} = 0{,}888 \tag{2.8}$$

In the equations for the individual periods only the equation for the years 1896—1914 had significant coefficients.

The results of the potential G.N.P. estimation by the equations (2.7)—(2.8) together with the general rate of resource utilisation are given in table 1. These results already take into account the corrections for the years with $\psi_{2+3} > 1$ For comparative purposes we present in table 2 American estimates of the potential G. N. P. based on the same data for the labor unemployment rate as our estimate by equation (2.8). Finally, in table 1 are also given the estimates of the capital and land utilisation rates on formula (2.5) and the above described correction procedure.

Table 1.

Potential G.N.P. and the rate of resurce utilisation in USA 1896—1966

	Calculations based on working time lost					Calculations based on unemployment rate of labor	
	Y_t^*	Y_t	ψ_t	ψ_{1t}	$\psi_{2+3,t}$	Y_t^*	ψ_t^*
	billons of $ 1958.					billons of $ 1958	
1896	70,4	60,0	0,850	0,876	0,792	67,1	0,893
1897	77,0	65,7	0,852	0,877	0,796	73,7	0,889
1898	77,4	67,1	0,866	0,886	0,818	74,3	0,902
1899	77,4	73,2	0,945	0,937	0,961	76,9	0,952
1900	79,5	75,3	0,945	0,937	0,959	78,0	0,964
1901	87,4	83,9	0,958	0,945	0,976	85,1	0,986
1902	87,2	84,7	0,972	0,955	1,000	86,2	0,983
1903	91,7	88,9	0,969	0,952	1,000	90,7	0,983
1904	93,1	87,8	0,942	0,935	0,956	90,6	0,968
1905	97,0	94,3	0,972	0,957	1,000	96,4	0,978
1906	109,3	105,2	0,962	0,947	0,981	105,6	0,996
1907	111,3	106,9	0,960	0,946	0,980	108,0	0,900
1908	112,2	98,1	0,873	0,891	0,845	108,7	0,902
1909	114,0	110,1	0,965	0,949	0,992	114,5	0,960
1910	113,3	111,3	0,982	0,971	1,000	117,3	0,948
1911	119,3	114,9	0,962	0,947	0,983	121,8	0,943
1912	122,5	120,3	0,981	0,968	1,000	125,1	0,960
1913	129,0	125,1	0,968	0,951	0,993	128,6	0,975

	Calculations based on working time lost					Calculations based on unemployment rate of labor	
	Y_t^*	Y_t	ψ_t	ψ_{1t}	$\psi_{2+3,t}$	Y_t^*	ψ_t^*
	billons of $ 1958					billons of $ 1958	
1914	131,7	115,5	0,876	0,893	0,844	126,7	0,911
1915	133,4	119,0	0,892	0,903	0,874	135,1	0,880
1916	137,8	135,7	0,985	0,973	1,000	140,1	0,968
1917	134,5	132,5	0,986	0,976	1,000	136,8	0,968
1918	148,5	144,5	0,974	0,957	1,000	146,5	0,991
1919	151,0	146,1	0,967	0,950	0,997	149,2	0,986
1920	148,4	144,4	0,973	0,958	1,000	148,0	0,975
1921	179,0	141,0	0,787	0,833	0,730	167,5	0,841
1922	177,9	149,3	0,838	0,878	0,778	162,6	0,918
1923	176,0	169,1	0,960	0,946	0,982	172,4	0,981
1924	190,1	174,1	0,915	0,918	0,911	182,1	0,955
1925	187,2	178,3	0,952	0,941	0,968	182,6	0,976
1926	199,4	189,9	0,952	0,941	0,969	193,4	0,989
1927	204,7	191,8	0,936	0,931	0,943	196,5	0,976
1928	206,5	194,1	0,939	0,933	0,948	198,9	0,973
1929	211,4	203,6	0,962	0,947	0,983	207,1	0,982
1930	199,7	183,5	0,918	0,920	0,915	204,2	0,898
1931	209,1	169,3	0,809	0,848	0,746	219,2	0,772
1932	223,3	144,2	0,645	0,727	0,503	224,3	0,642
1933	243,1	141,5	0,581	0,674	0,421	227,5	0,621
1934	236,1	154,3	0,653	0,733	0,521	228,9	0,673
1935	245,4	169,5	0,690	0,762	0,576	241,9	0,700
1936	253,6	193,0	0,760	0,814	0,678	255,7	0,754
1937	251,0	203,2	0,809	0,848	0,748	254,0	0,799
1938	269,0	192,9	0,717	0,782	0,611	268,2	0,719
1939	276,7	209,4	0,756	0,811	0,667	279,3	0,749
1940	280,6	227,2	0,809	0,848	0,750	285,9	0,794
1941	297,6	263,7	0,885	0,899	0,865	300,5	0,877
1942	311,1	297,8	0,957	0,944	0,979	307,1	0,969
1943	345,1	337,1	0,977	0,964	1,000	351,2	0,988
1944	370,0	361,3	0,977	0,964	1,000	364,2	0,992
1945	364,4	355,2	0,975	0,962	1,000	360,0	0,988
1946	325,4	312,6	0,960	0,946	0,986	320,5	0,975
1947	322,6	309,9	0,960	0,946	0,984	317,3	0,976
1948	371,8	323,7	0,870	0,889	0,838	331,9	0,976
1949	340,3	324,1	0,952	0,941	0,970	341,7	0,948
1950	374,9	355,3	0,947	0,938	0,961	370,4	0,959
1951	399,1	383,4	0,960	0,946	0,984	391,4	0,979
1952	410,6	395,1	0,962	0,947	0,989	403,0	0,981
1953	430,5	412,8	0,958	0,945	0,983	419,5	0,981

	Calculations based on working time lost					Calculations based on unemployment rate of labor	
	Y_t^*	Y_t	ψ_t	ψ_{1t}	$\psi_{2+3,t}$	Y_t^*	ψ_t^*
	billons of $ 1958					billons of $ 1958	
1954	433,2	407,0	0,939	0,933	0,949	425,9	0,955
1955	467,7	438,0	0,936	0,931	0,945	450,6	0,972
1956	477,2	446,1	0,934	0,930	0,943	458,0	0,974
1957	485,7	452,5	0,931	0,928	0,936	465,0	0,973
1958	491,8	447,3	0,909	0,914	0,898	480,0	0,932
1959	521,5	475,9	0,912	0,916	0,906	498,0	0,955
1960	534,4	487,7	0,912	0,916	0,904	510,3	0,955
1961	554,3	497,2	0,896	0,906	0,877	532,2	0,934
1962	587,6	529,8	0,901	0,909	0,885	554,4	0,955
1963	612,2	551,0	0,899	0,908	0,882	578,7	0,952
1964	642,2	580,0	0,903	0,910	0,891	600,7	0,966
1965	676,8	614,4	0,907	0,913	0,897	634,0	0,970
1966	716,4	652,6	0,910	0,915	0,901	669,9	0,975

Table 2

A Comparison of results with the estimations made in USA

	Black-Russel	Okun	Phelps[1]	Council of Economic Advisers	Our estimate
		billons of $ 1958			
1953	—	—	412,8	—	419,5
1954	—	—	423,5	—	425,9
1955	433,0	443,4	444,0	438,8	450,6
1956	447,7	448,5	451,0	454,1	458,0
1957	464,3	457,4	458,0	470,0	465,0
1958	479,6	488,1	490,6	486,4	480,0
1959	495,3	498,6	493,7	503,0	498,0
1960	514,9	512,7	505,0	521,1	510,3
1961	536,8	540,2	537,0	539,3	532,2
1962	556,6	556,6	544,0	558,0	554,4
1963	576,9	580,6	—	578,6	578,7
1964	598,4	602,5	—	602,5	600,7
1965	620,7	625,2	—	622,6	634,0

[1]) Reevaluated in 1958 dollars with the help of a general rate of resource utilisation, i.e. by the ratio between actual and potential G. N. P. in 1954. dollars.

These results lead to the following conclusions:

1) The estimates of potential G.N.P. sharply contrast in the dependence on the assumed conception: the purely economic concept of production possibilities (potential G.N.P. evaluation using unemployment rate of labor), the purely technical one (the calculation of technically feasible potential G.N.P.) and the intermediate concept of the potential G.N.P. estimation with the help of working time lost under the constant traditional number of shifts. It seems, that just the latter conception gives an acceptable compromise between inconsistent extreme definitions of the production possibilities and potential G.N.P.

2) The above described algorithm and computer program give the possibilities of estimation of both potential G.N.P., the utilisation rates of separate primary resources, and the simple procedure of reestimation secures the consistency of the results. The application of the algorithm did not depend upon the assumed concept of the potential G.N.P.

3) A comparison of different methods (based on the same data) indicates fairly good accuracy of our results. Their shortcoming consists in the impossibility of improving the accuracy by the use of relatively short time series.

4) In the presence of an adaptive economic mechanism and with corresponding statistical data this method can be used for any economy. The scope and perspectives of its application will be much greated when an iterative procedure will be created for coordination of the potential G.N.P. estimates for a fairly detailed list of industries because this directly leads to a consistent system of estimates of industrial capacities.

APPENDIX 1

A short description of the computer program.

This program is a modification of the more general computer program for the estimation of a multilinear (or one reducible to such form) regression with autoregressive tranformation of the first order and a confidence regions evaluation.

The sequence of the computation is is the following:

1. The parameters of the main equation:

$$\ln Y_t = a_o + a_1 + b_o t + c_o t^2 + d_o t^3 + f_o t^4 + b_1 u_t + c_1 u_t^2 \qquad (1)$$

are evaluated together with σ, σ^*, v, t^* and t^*-statistics for each coefficient. The program is fitted for the estimation of the equation (1) if $T \leq 75$, i.e. the number of observation does not exceed 75.

2. From the condition

$$exp\,(a_1 + 3\,b_1 + 9\,c_1) = 1 \qquad (2)$$

is estimated a_1.

3. For each year are computed:

$$Y^*_{t} = Y_t\,[\exp\,(a_1 + b_1 u_t + c_1 u^2_t)]^{-1} \qquad (3)$$

$$\psi_t = Y_t / Y^*_t \qquad (4)$$

For (4) are estimated the mean, standard deviation, coefficient of variation, i. e. $\overline{\psi}_t$, $\sigma\psi_t$, $v\psi_t$

4. The rates of growth of the actual and potential G.N.P. are computed together with their means. standard deviations and coefficients of variation:

$$g_{Y_t} = \frac{Y_{t+1} - Y_t}{Y_t},\ \bar{g}_{Y_t},\ \sigma g_{Y_t},\ v g_{Y_t} \qquad (5)$$

$$g_{Y^*_t} = \frac{Y^*_{t+1} - Y_t}{Y^*_t},\ \bar{g}_{Y^*_t},\ \sigma g_{Y^*_t},\ v g_{Y^*_t} \qquad (6)$$

5. Then follows the calculation

$$\psi_{2+3,t} = \frac{\psi_t - \alpha_{1t}\,\psi^*_{1t}}{\alpha_{2t} + \alpha_{3t}} \qquad (7)$$

For the years with $\psi_{2+3t} > l$ it is assumed $\psi_{2+3,t} = 1$ and the reestimation of ψ_t for these years is made:

$$\psi_t^*\ \alpha_{1t}\ \psi_{1t} + (\alpha_{2t} + \alpha_{3t}) \qquad (8)$$

6. Finally, a reestimation of the expression (6) follows:

$$g_{Y^*_t}{}^{1} = \frac{Y^{*1}_{t+1} - Y_t^{*1}}{Y_t^{*1}} = \frac{Y_{t+1} / \psi^*_{t+1} - Y_t / \psi^*_t}{Y_t / \psi^*_t} \qquad (9)$$

The output includes: *a*

1. a_0, b_0, c_0, d_0, f_0, b_1, c_1 together with their t*-statistics.
2. $ln\ Y_t$ calculated, σ, α^*, v, a_1.
3. Correlation matrix r_{ij}.
4. Y_t^*, ψ_t, $\overline{\psi}_t$, $\sigma\psi_t$, $v\psi_t$, g_{Y_t}, $\sigma_{Y^*_t}$, σg_{Y_t}, $\sigma g_{Y_t^*}$ $v g_{Y_t}$, $v g_{Y_t^*}$
5. $\psi_{2+3,t}$, ψ_t^*, Y_t^{*1}, $g_{Y_t^{*1}}$, $\sigma g_{Y_t^{*1}}$, $v g_{Y_t^{*1}}$.

APPENDIX 2.

Working time losses in the USA 1896—1966

	Part-unemployment (% of working time)	Unemployment rate of labor	Equivalent of unemployment rate of labor	Total working time lost (%)
1896	9,0	9,0	3,4	12,4
1897	8,8	9,2	3,5	12,3
1898	8,4	8,5	3,0	11,4
1899	4,8	5,7	1,5	6,3
1900	5,0	5,0	1,3	6,3
1901	5,1	2,4	0,4	5,5
1902	4,1	2,7	0,4	4,5
1903	4,4	2,6	0,4	4,8
1904	5,3	4,8	1,2	6,5
1905	3,6	3,1	0,7	4,3
1906	5,1	0,8	0,2	5,3
1907	5,1	1,8	0,3	5,4
1908	7,9	8,5	3,0	10,9
1909	3,7	5,2	1,4	5,1
1910	1,3	5,9	1,6	2,9
1911	3,2	6,2	2,1	5,3
1912	1,8	5,2	1,4	3,2
1913	3,8	4,4	1,1	4,9
1914	6,0	8,0	4,7	10,7
1915	5,8	9,7	3,9	9,7
1916	1,5	4,8	1,2	2,7
1917	1,2	4,8	1,2	2,4
1918	4,1	1,4	0,2	4,3
1919	4,6	2,3	0,4	5,0
1920	3,0	4,0	1,2	4,2
1921	8,0	11,9	8,7	16,7
1922	2,0	7,6	11,2	13,2
1923	2,0	3,2	3,4	5,4
1924	6,0	5,5	2,2	8,2
1925	3,0	4,0	2,9	5,9
1926	3,0	1,9	2,9	5,9
1927	4,0	4,1	2,9	6,9
1928	3,0	4,4	3,7	6,7
1929	4,6	3,2	0,7	5,3
1930	4,9	8,7	3,1	8,0

	Part-unemployment (% of working time)	Unemployment rate of labor	Equivalent of unemployment rate of labor	Total working time lost (%)
1931	6,0	15,9	9,2	15,2
1932	7,3	23,6	20,0	27,3
1933	10,3	24,9	22,3	32,6
1934	9,6	21,7	17,1	26,7
1935	9,1	20,1	14,7	23,8
1936	8,0	16,9	10,6	18,6
1937	7,4	14,3	7,8	15,2
1938	8,5	19,0	13,3	21,8
1939	7,9	17,2	11,0	18,9
1940	7,1	14,6	8,1	15,2
1941	6,1	9,9	4,0	10,1
1942	4,4	4,7	1,2	5,6
1943	3,3	1,9	0,3	3,6
1944	3,4	1,2	0,2	3,6
1945	3,5	1,9	0,3	3,8
1946	4,4	3,9	1,0	5,4
1947	4,6	3,9	0,8	5,4
1948	10,4	3,8	0,7	11,1
1949	4,7	5,9	1,2	5,9
1950	4,9	5,3	1,3	6,2
1951	4,7	3,3	0,7	5,4
1952	4,8	3,8	0,5	5,3
1953	5,0	2,9	0,5	5,5
1954	5,4	5,5	1,3	6,7
1955	5,7	4,4	1,2	6,9
1956	6,0	4,1	1,0	7,0
1957	6,2	4,3	1,0	7,2
1958	6,6	6,8	2,0	8,6
1959	6,7	5,5	1,7	8,4
1960	6,9	5,5	1,5	8,4
1961	7,2	6,7	2,2	9,4
1962	7,4	5,5	1,7	9,1
1963	7,5	5,7	1,7	9,2
1964	7,6	5,2	1,4	9,0
1965	7,6	4,5	1,1	8,7
1966	7,7	3,8	0,8	8,5

APENDIX 3.

Estimation α_{1t} for the period 1896—1966 for US economy

The process consists of two stages: 1) an estimation of the national income in current prices, 2) an estimation of the incomes imputed to labor force.

1. An estimation of national income in current prices

For the period 1929—1966 there are available data (25—26). For the years 1896—1929 the G.N.P. volume in 1929 prices in the Department of Commerce concept was derived in (28, p. 251—254). Using the same data the G.N.P. deflator can be calculated. As an estimate for depreciation the figures in (29) were taken, and they were controlled by the data in (30) on depreciation in current prices. Then the data on national income in current prices for the whole period 1896—1966 were derived by a simple substraction of depreciation at current prices from G.N.P. at current prices.

2. The calculation of the income imputed to the labor force

The data in (28) embraces the whole U.S. economy 1899—1953 with a separation of the private sector and agriculture. Further, there were data in (31) concerning the incomes of employees in the private economy during the years 1899—1929. After 1929 the data on the income of employees for the U.S. economy can be found in ready form in (25—26).

Therefore, it was necessary to put in comparable form the data on the incomes of employees and to separate from total farm income the part imputed to the labor force.

During the first operation it was necessary to find the income of employees in the government sector, including military personnel, for the period 1899—1929, thus to estimate the total income of employees, and then to add the data for the years 1896—1898.

The income of employees in the government sector was derived in the following way. The ratio between the number of man-hours of employees in the government and private sectors as a starting point. This, however, implied the exclusion from the data (28, p. 262) of the number of man-hours of family workers, mainly in agrictulture. This ratio for agriculture during 1909—1929 was calculated using data in (32, 1962), and for the years 1890, 1900, 1909, according to the ratio of farmers and family workers with the following interpolation: Summing up the numbers of man-hours of employees in the non-agricultural sector and agricultural family workers, it is possible to estimate the ratio of man-hours of employees in the government and private sectors.

In order to take into account the difference in the salaries of employees in the government and private sectors we used the direct estimates for the years 1913—1927 (33, p. 476) and the data from the forties (34). All these data indicate that the difference was something like 1/5. For 1899 and

1909 the ratio was indirectly controlled using Census of Manufactures data on wage-bills.

Thus it was possible to evaluate the income of employees in the government sector and then the total income of employees during 1899—1929.

The income of employees for the years 1897—1898 was calculated from 1899 as a starting point, using the data (24, p. 455) on employment in industry, construction and transport together with the index of the yearly wage of the full and part — time employees (27, s. 32) and the above described scheme for the income estimation of employees in the government sector.

The estimation of the farming income imputed to the labor force α_{1t} was carried out in the following way:

1. The national income originating in agriculture in 1929 prices was calculated with the help of the data on agricultural G.N.P. in 1929 prices (28, p. 255), service life and volume of agricultural capital (33) (these latter data were necessary for the estimation of depreciation). The agricultural national income for 1896—1929 in 1929 prices was then converted te current prices. Further, on the basis of the data (28, p. 247) on the income share of the labor force in agricultural national income, the total sum of labor's income was distributed between farmers and employees roughly proportional to the share of family workers, and for some years the indirect control through the sum of wages and salaries was carried out.

2. For the period 1929—1966 the calculation was carried out on the basis of the available data (25—26), the information concerning the wage-bill of agricultural employees during 1947—1965 (34) and the share of labor income equal to 0,70 of the agricultural national income. In the rest the sheme from the years 1897—1929 was repeated.

3. Thus derived agricultural incomes imputed to the labor force were added to the incomes of employees, although between time-series 1896—1929 and 1929—1966 remains some difference due to the inclusion of the supplements to wages beginning in 1929 (equal in that year to 1,31% of the wage-bill).

The results of this procedure are given in the table below.

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Income imputed to the labor force in USA α_{1t} 1896—1966 (current prices)

Years	Income of emfloyees in private economy	Total income of employees	Total income of labor force	Total national income	α_{1t}
	(billons of $)				
1896	—	—	8,34	12,058	0,692
1897	7,02	7,47	9,16	13,335	0,686
1898	7,48	7,96	9,92	14,007	0,708
1899	7,72	8,21	10,17	15,774	0,645
1900	8,26	8,81	10,79	16,978	0,635
1901	8,91	9,48	11,17	18,935	0,590
1902	9,79	10,40	12,51	19,714	0,634
1903	10,55	11,21	13,26	20,945	0,633
1904	10,77	11,50	13,73	20,856	0,658
1905	11,85	12,64	14,85	22,981	0,644
1906	12,83	13,68	15,01	26,337	0,570
1907	13,49	14,41	16,60	27,770	0,598
1908	12,07	12,96	15,52	25,044	0,620
1909	13,92	14,95	17,88	28,279	0,632
1910	14,90	16,04	18,99	30,379	0,625
1911	15,01	16,17	18,22	31,162	0,585
1912	15,88	17,13	20,29	34,031	0,596
1913	17,18	18,55	20,76	35,565	0,584
1914	16,75	18,25	21,07	32,880	0,640
1915	17,40	18.95	21,69	34,974	0,620
1916	20,88	22,61	25,35	45,485	0,558
1917	24,58	28,50	33,05	54,374	0,608
1918	30,02	37,00	42,01	68,981	0,609
1919	34,47	39,75	45,11	71,023	0,634
1920	41,76	46,15	51,30	80,014	0,641
1921	32,30	35,60	37,40	66,821	0,560
1922	33,71	37,20	40,31	67,193	0,600
1923	40,35	44,50	47,88	78,579	0,609
1924	40,02	44,30	47,58	80,005	0,595
1925	41,43	45,87	49,82	83,576	0,596
1926	44,37	48,95	52,39	89,343	0,586
1927	44,37	49,17	52,71	87,980	0,599
1928	45,13	49,90	53,58	89,652	0,598
1929	45,50	50,44	54,85	95,227	0,576
1930	—	46,85	49,62	82,394	0,602
1931	—	39,75	42,02	67,962	0,618
1932	—	31,06	32,62	50,680	0,636

Years	Income of employees in private economy	Total income of employees	Total income of labor force	Total national income	α_{1t}
	(billons of $)				
1933	—	29,55	31,30	48,648	0,634
1934	—	34,30	36,23	58,220	0,622
1935	—	37,35	40,25	65,374	0,616
1936	—	42,91	45,50	75,444	0,603
1937	—	47,93	51,30	83,289	0,616
1938	—	45,00	48,10	77,379	0,622
1939	—	48,11	51,25	83,227	0,616
1940	—	52,13	55,34	92,175	0,600
1941	—	67,78	69,18	116,320	0,594
1942	—	85,26	91,64	148,114	0,618
1943	—	109,55	117,22	181,342	0,646
1944	—	121,21	129,03	199,080	0,648
1945	—	123,10	133,31	200,682	0,664
1946	—	117,85	127,78	198,620	0,643
1947	—	128,89	139,01	219,104	0,634
1948	—	141,13	154,56	243,072	0,636
1949	—	141,03	150,57	239,934	0,628
1950	—	154,57	163,67	266,426	0,614
1951	—	180,69	192,78	307,209	0,627
1952	—	195,31	206,68	322,306	0,640
1953	—	209,11	218,90	338,920	0,646
1954	—	207,96	217,04	336,607	0,645
1955	—	224,48	232,79	366,486	0,635
1956	—	243,06	251,40	385,167	0,652
1957	—	256,00	264,26	404,045	0,654
1958	—	257,82	267,81	408,396	0,656
1959	—	279,09	287,54	442,285	0,650
1960	—	294,23	303,20	460,326	0,658
1961	—	302,64	312,30	474,853	0,658
1962	—	323,63	333,50	510,355	0,653
1963	—	341,00	350,99	537,902	0,653
1964	—	365,66	375,13	575,664	0,652
1965	—	393,93	405,53	621,618	0,652
1966	—	435,72	447,52	679,782	0,658

REFERENCES

1. G. Debreu, »The Coefficient of Resources Utilisation«, *Econom.*, v. 19, N 1, 1951.
2. J. Lesourne, *Critères de Comparison des Situations Economiques*, Congres d'Econometrie de Bilbao, 1958 (mimeogr.).
3. R. Lipsey, K. Lancaster, »The General Theory of Second Best«, *Rev. of Econ. Stud.*, v. 24, N 63, 1956—1957.
4. A. Harberger, »The Measurement of Waste«, *Amer. Ec. Rev.*, v. 44, N 3, 1964, Pap. and Proceed.
5. В. Леонтьев, Исследования структуры американской экономики, М., Госстатиздат, 1959.
6. L. Klein, »Some Theoretical Issue in the Measurement of Capacity«, *Econom.*, v. 28, N 2, 1960.
7. L. Klein, R. Preston, »Some New Results in the Measurement of Capacity Utilisation«, *Amer. Econ. Rev.*, v. 57, N 1, 1967.
8. M. Foss, »The Utilisation of Capital Equipment: Postwar Compared with Prewar«, *Survey of Current Business*, v. 43, N 6, 1963.
9. D. Jorgenson, Z. Griliches, »The Explanation of Productivity Change«, *Rev. of Ec. Stud.*, v. 34(4), N 100, 1967.
10. N. Choudry, K. Mohabbat, »Economic Fluctuations and Growth: Notes on Testing the Smithies Model«, *Oxford Econ. Papers*, v. 17, N 1, 1965.
11. A. Okun, »Potential G.N.P.: Its Measurement and Significance«, *Amer. Stat. Assoc., Proceed.*, 1962, p. 98—104.
12. E. Phelps, Ch. Phelps, »Factor-Price Frontier Estimation of Vintage Production Model of the Postwar U.S. Non-farm Business Sector«, *Rev. of Ec. and Stat.*, v. 58, N 3, 1966.
13. S. Black, R. Russell, »An Alternative Estimate of Potential G.N.P.«, *Rev. of Ec. and Stat.*, v. 60, N 1, 1969.
14. L. Klein, R. Summers, *The Warton Index of Capacity*, Phil. 1968.
15. F. de Leeuw, »A Revised Index of Manufacturing Capacity«, *Federal Reserve Bul.*, Dec. 1966.
16. R. Solow, »Technical Progress, Capital Formation and Economic Growth«, *Amer. Ec. Rev.*, v. 53, N 2, 1962.
17. M. Brown, *On the Theory and Measurement of Technological Change*, Camb. 1966.
18. D. Streever, »Capacity Utilisation and Business Investment«, *Univ. of Illinois, Bureau of Ec. and Bus. Research, Bul.* N 86, 1960.
19. A. Nourse, *America's Capacity to Produce*, Wash. 1939.
20. Office of Military Government for Germany (US) *Statistisches Handbuch von Deutschland 1928—1944*, T. III A, München, 1946.
21. С. Вишнев, Экономические ресурсы Германии, М., Соцэкгиз, 1940.
22. U.S. Governement Printing Office, *Historical Statistics of the United States from Colonial Times to 1957*, Wash., 1960.
23. *Statistical Abstract of the United States.*
24. P. Douglas, *Real Wage in the United States 1890—1926*, N.Y., 1930.
25. U.S. Department of Commerce. *The National Income and Product Accounts of the United States 1929—1965.*
26. *Survey of Current Business*, July 1967.

27. J. Kuczynski, *Die Geschichte der Lage der Arbeiter in den Vereinigten Staaten von Amerika seit 1898*, B. 1955.
28. Д. Кендрик, Тенденции производительности в США, М., Прогресс, 1967.
29. S. Kuznets, *Capital in the American Economy: Its Formation and Financing*, N.Y. 1961.
30. R. Goldsmith, *A Study of Savings in the United States, vol. III, Special Studies, Princeton*, 1956.
31. A. Grant, »Issues in Distribution Theory: The Measurement of Labor's Relative Share«, 1899—1929, *Rev. of Econ. and Stat.*, v. 45, N 3, 1963.
32. U.S. Department of Agriculture, *Agricultural Statistics.*
33. Р. Голдсмит, Национальное богатство США в послевоенный период, М., Прогресс, 1968.
34. J. Gottsegen, »Revised Estimates of G.N.P. by Major Industries«, *Survey of Current Business*, 1967, N 4.

DYNAMIC PROGRAMMING AS A POTENTIAL PLANNING TOOL

1. The Scope of the Paper

This paper attempts to present some facts and examples relevant to the claim that dynamic programming is a potentially useful planning tool. I say »potentially« because first of all I am not entirely convinced that it is useful, although the more experience I gain the more convinced I become, and secondly dynamic programming has such a short history of application in economic planning that evidence on this point is not plentiful. True, such venerable researchers as Stone and his colleagues (1) made recourse to it briefly during the early stages of the Cambridge Growth Project, but this by itself hardly justifies the case for dynamic programming.

The important thing when investigating a new tool or technique is to obtain plenty of practical experience by using it. As Nemhauser has said, »It is absolutely necessary to solve problems to understand dynamic programming« (2). Over the past fifteen months, I have been endeavouring to achieve just this aim by applying dynamic programming to some of the problems which have occurred during research being carried out by the National Economic Planning Unit at Birmingham University. The main emphasis of this research, as several publications show (3), (4), (5), lies in the field of large optimization models, which are being solved using various decompositional techniques. As we shall see later, dynamic programming becomes rather inefficient when applied to large systems, generally losing out to more established techniques.

One cannot help getting the feeling when using dynamic programming, however, that such an inherently usable approach must have advantages for certain planning problems. The ease with which it can be used to obtain solutions to problems for which previously only approximations were possible quickly convinces the planner that there is something worth investigating here. Dangers loom large for the researcher who unwittingly strays too close to the boundaries of reasonability, or the user who expects too much too soon, and such hazards may be sufficient to persuade the impatient that no further effort is justified. This is not the opinion of this writer.

I hope to show, with the aid of three simple examples, that there are grounds for continuing to devote time and effort to the study of dynamic programming as a possible planning tool. This problem solving approach is supported by no less an authority than Bellman, the father and discoverer of dnamic programming, who said, »At the present time, we possess a number of powerful mathematical techniques for the analytic and computational solution of classes of problems in the field of mathematical economics. What is now needed is a systematic exploitation of these methods to provide a backlog of solved problems which will guide our subsequent research« (6).

Since this statement, many applications have been made, but the majority seem to have been in fields other than economics. It is hoped that this paper will redress the balance slightly by looking at three problems from inventory, allocation and capital budgeting. The approach throughout is simplified where possible so as not to confuse the non-mathematical planner, although we are doing our best at Birmingham to eliminate this species. In the inventory problem especially, this means looking at a problem usually as stochastic-continuous in a deterministic-discrete way. By so doing, frequency functions and the like are avoided, and the essential features of dynamic programming are that much clearer.

The more discerning planner might also criticise the models in one or two cases. I would probably support him were that the purpose of this paper, but it is not. The approach is to be considered above all else, and discussions of the models must await another opportunity. When speaking a foreign language, it is reassuring to know at least something about the topic of conversation, so that the learner can concentrate on the grammar; the models, are, therefore, familiar and uncomplicated.

One more point which must be made is that I have some qualms about my passport being valid for the country in which I find myself. Economica contains far too many regions where the dialect is incomprehensible to a simple mathematician. I trust that the reader will forgive the odd crude phrase here and there, and that my strange turn of tongue allows the essentials nevertheless to come through.

Before beginning the treatment of the three examples, a few introductory remarks on dynamic programming in general are probably in order. Firstly, the term itself is often misleading to the casual reader. Perhaps recursive optimization or sequential programming would better describe what is actually going on. The problem being considered need contain no reference to time, as »dynamic« might imply, although many of the planning problems suitable for treatment are naturally of this type.

To take the novice further, the next words he must learn are stage, state, return, and decision. The following diagram contains all four, and speaks for itself: