

zemalja, u svim se socijalističkim zemljama i makroagregatima fluktuacije smanjuju s vremenom. To je bilo utvrđeno na osnovu upoređivanja koeficijenata fluktuacije za razdoblje 1955—65. sa razdobljem 1950—60. god.

Vrlo su interesantni rezultati izračunavanja prosečnih stopa rasta i njihovih trendova. Stopa rasta u socijalističkim zemljama su prilično visoke, više nego u kapitalističkim zemljama. Međutim, sa jedinim izuzetkom Jugoslavije, one pokazuju u razdoblju 1950—65. god. negativan trend, znači — smanjuju se.

Svi podaci iz kojih proizlaze gornji rezultati su službeni podaci odnosnih zemalja, objavljeni velikom većinom u odnosnim statističkim godišnjicima. Autor nije tretirao pitanje u koliko meri ti podaci odgovaraju realnosti ni da li su uporedivane stope suštinski uporedive.

#### AJUSTIRANJE, INTERPOLACIJA I EKSTRAPOLACIJA SEZONSKIH VREMENSKIH SERIJA

*Branislav IVANOVIC*

1. Ako se u toku vremenskog razmaka od  $N$  godina, svake godine meri  $n$  puta obeleže  $X$  i ako je  $x_{ij}$  njegova  $j$ -ta vrednost u  $i$ -toj godini, matrica

$$X = \begin{vmatrix} x_{11} & \dots & x_{1n} \\ \dots & \dots & \dots \\ x_{N_1} & \dots & x_{Nn} \end{vmatrix}$$

predstavljaće tada niz sezonskih vremenskih serija. Varijacije između podataka jedne iste godine i između odgovarajućih vrednosti podataka različitih godina mogu biti posledica aleatornog karaktera, sezonskog karaktera i opšte tendencije u razvoju posmatrane pojave. Ako je dovoljno dug posmatrani vremenski razmak, ciklički karakter pojave može se takođe afirmisati u opštoj tendenciji razvoja.

Ako je u pojavi jako naglašen sezonski karakter i ako su međusezonske varijacije više manje stabilne, relativne izravnate vrednosti u  $(t+1)$ -oj godini biće

$$(1.1) \quad \theta_{t+1,t}^* = \sum_{j=t-1}^{t+1} k_{jt} \theta_{tj},$$

gde je

$$i \in \{1, \dots, n\}, \quad t \in \{1, \dots, N-1\},$$

$$i = 1 \longrightarrow i-1 = n \text{ i } \theta_{t,t-1} = \theta_{t-1,n},$$

$$i = n \longrightarrow i+1 = 1 \text{ i } \theta_{t,t+1} = \theta_{t+1,1}.$$

a koeficijentii  $k_{ji}$  su elementi zakona evolucije

$$(1.2) \quad K = \begin{bmatrix} k_{11} & k_{21} & 0 & 0 & \dots & k_{n1} \\ k_{12} & k_{22} & k_{32} & 0 & \dots & 0 \\ 0 & k_{23} & k_{33} & k_{43} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & k_{n-2,n-1} & k_{n-1,n-1} & k_{nn} \\ k_{1n} & 0 & \dots & 0 & k_{n-1,n} & k_{nn} \end{bmatrix}$$

čije su vrednosti dobijene pomoću metode najmanjih kvadrata

$$(1.3) \quad \vec{k}_i = \vec{\beta}_i^{i-1}, \quad i \in \{1, \dots, n\},$$

a elementi matrice i vektora  $\vec{\beta}_i$  dati su preko obrazaca

$$(1.4') \quad \alpha_{ij} = \sum_{t=1}^{N-1} \theta_{it} \theta_{jt}, \quad \langle s, j \rangle \leq \{i-1, i, i+1\}$$

$$(1.4'') \quad \beta_{ij} = \sum_{t=1}^{N-1} \theta_{i+t, i} \theta_{jt}, \quad i \in \{i-1, i, i+1\}$$

Predviđanje budućih sezonskih vremenskih serija koje neposredno sledi posmatrani vremenski period, vrši se preko relacija

$$(1.5) \quad \hat{\theta}_{Ni} = \theta_{Ni}$$

$$\hat{\theta}_{N+r, i} = \sum_{j=l-1}^{l+1} k_{ji} \hat{\theta}_{N+r-l, j}, \quad r \in N, \quad i \in \{1, \dots, n\}$$

dok relacija

$$(1.6) \quad \theta_{it} = \sum_{j=i-1}^{i+1} c_{ji} \theta_{i+j, t}, \quad t \in E_-, \quad i \in \{1, \dots, n\}$$

sa

$$(1.7) \quad \vec{\alpha} = \vec{\beta}_i^i \quad \begin{bmatrix} a_{i-1, i-1} & a_{i, i-1} & a_{i+1, i-1} \\ a_{i-1, i} & a_{ii} & a_{i+1, i} \\ a_{i-1, i+1} & a_{i, i+1} & a_{i+1, i+1} \end{bmatrix}$$

i

$$(1.8) \quad a_{ij} = \sum_{t=1}^{N+1} \theta_{i+t, i} \theta_{i+t, j}, \quad \langle s, j \rangle \leq \{i-1, i, i+1\}$$

omogućava da se izvrši rekonstrukcija onih sezonskih vremenskih serija koje prethode posmatrani vremenski period.

Ako su intersezonske varijacije zanemarljive u odnosu na slučajne varijacije pojave, nećemo morati više da zadržavamo sezonsku komponentu varijacije u zakonu evolucije. Zato će sada zakon evolucije biti oblika

$$(1.9) \quad K = [\theta_{ij}] [\alpha_{ij}]^{-1}$$

a odgovarajući niz ažustiranih sezonskih vremenskih serija biće

$$(1.10) \quad \theta^* = \theta K.$$

Najzad, preko relacija

$$(1.11) \quad \hat{\theta}_{Ni} = \theta_{Ni}$$

$$\hat{\theta}_{N+r, i} = \sum_{j=1}^n k_{ji} \hat{\theta}_{N+r-1, j}, \quad r \in N, \quad i \in \{1, \dots, n\}$$

možemo vršiti predviđanja budućih i preko

$$(1.12) \quad \hat{\theta}_{it} = \theta_{it}$$

$$\hat{\theta}_{it} = \sum_{j=1}^n k_{ji}^{-1} \theta_{i+j, t}, \quad t \in E_-, \quad i \in \{1, \dots, n\}$$

prethodne sezonske vremenske serije.

Ovaj postupak za ažustiranje i predviđanje sezonskih vremenskih serija nazvali smo G-metodom pri čemu je G-1 varijanta u kojoj se ne uzima u obzir sezonski karakter pojave a G-2 varijanta koja uključuje sezonsku komponentu u zakon evolucije.

Primenimo još da je za primenu varijante G-1 potrebno da broj godina u posmatranom periodu ne bude manji od broja sezona, tj. ne sme biti manji od četiri ako su podaci kvartalni niti manji od dvanaest ako su podaci mesečni. Uopšte, u početnoj matrici X broj redova ne sme da bude manji od broja kolona.

2. Kao primer uzeli smo izvoz Velike Britanije u vremenskom periodu od 1960. do 1967. god. zaključno. Tabela (1) daje mesečne podatke toga izvoza, izražene u milionima funti sterlinga. Posmatrajući mesečne proseke ( $\bar{x}$ ) možemo zaključiti da je izvoz Velike Britanije bio značajniji s proleća i pri kraju godine dok je u avgustu i septembru bio najslabiji. Upoređenja radi, primenimo dobro poznatu Washingtonsku metodu — varijantu X-11 — za izravnjanje datog niza vremenskih serija. Dobijeni rezultati su prikazani u tabeli (2). Odmah se može zapaziti da su mesečni proseci sada vrlo bliski i da se, prema tome, izgubio sezonski karakter pojave. Takođe vidimo da je variabilitet po kolona-

ma izravnatih vrednosti ostao visok i da je kod šest 'kolona (januar, april, juni, juli, avgust i septembar) viši nego kod originalnih podataka. Ako sada primenimo varijantu G-1, dobićemo izravnate vrednosti koje su date u tabeli (3). Kako varijanta G-1 zadržava sezonski karakter pojave to vidimo da su i izravnate vrednosti u proletnim mesecima kao i u novembru i decembru jače izražene. Varijabilitet se smanjio i samo je u aprilu i avgustu nešto viši nego kod originalnih vrednosti.

Tabela (4) odnosi se takođe na izvoz Velike Britanije i na isti vremenski razmak s razlikom što su sada podaci tromesečni. U tabeli (5) date su izravnate vrednosti pomoću metode X-11 a u tabelama (6) i (7) izravnate vrednosti preko varijanata G-1 i G.

3. Posle izvršene analize dobijenih rezultata, Statistički ured Ujedinjenih nacija odlučio je da se u publikacijama sezonskih vremenskih serija svetskog uvoza i izvoza, koje će ubuduće redovno objavljivati Centar za statistiku spoljne trgovine UN, primenjuje metoda G za izravanjanje posmatranih i predviđanje budućih serija. U tom cilju izdat je nalog elektronskom centru UN da napravi plan za izvršenje svih potrebnih priprema i razradu Fortranovog programa metode G. Na projektovanju toga plana učestvovali su Sidney Cashton, direktor Elektronskog centra, i William Mackay, šef odeljenja za analizu sistema i programiranja, a definitivan program za obe varijante metode G obradio je Vicent Dumitru, programer Elektronskog centra UN. Dok je za varijantu X-11 Vašingtonske metode potrebno oko pet stotina strana Fortranovog programa, doatile program za obe varijante metode G iznosi svega deset strana.

God.	Izvoz Velike Britanije 1960—1967 (u milionima funti sterlina)												TOTAL
	Jan.	Feb.	Mart	Apr.	Maj	Juni	Juli	Avg.	Sept.	Okt.	Nov.	Dec.	
1960.	306.6	298.1	291.9	299.1	285.9	284.9	276.8	293.9	295.1	297.1	299.8	307.2	3535.4
1961.	310.0	306.7	304.0	309.2	310.2	315.2	314.2	314.3	300.0	306.0	320.8	301.5	3712.0
1962.	310.5	307.2	304.7	321.1	309.4	320.6	318.4	310.3	324.4	317.5	304.3	317.3	3765.8
1963.	319.3	321.7	342.4	317.4	347.8	338.3	345.6	340.0	346.0	350.7	344.4	348.7	4062.4
1964.	349.5	353.7	354.9	351.7	369.3	343.9	339.2	363.0	336.4	361.8	353.3	372.3	4248.9
1965.	368.3	371.4	376.3	403.2	370.2	407.9	421.0	407.3	416.8	390.6	415.3	421.3	4769.5
1966.	397.4	413.0	402.1	388.3	399.3	392.4	396.5	427.2	439.3	446.8	473.1	423.9	4999.4
1967.	444.4	448.0	434.6	438.6	438.7	440.7	430.1	393.8	407.9	394.3	367.1	377.8	5016.1
$\bar{x}$	347.7	349.3	376.5	357.0	376.4	350.6	352.6	335.8	320.1	351.5	376.5	370.4	
$\sigma$	40.6	54.7	58.4	44.0	53.5	48.3	51.3	40.4	52.1	50.2	58.9	56.9	

Tabela 1.

Tabela 2.

Izravnate vrednosti izvoza Velike Britanije  
(pomoću metode X-11)

God.	Jan.	Feb.	Mart	Apr.	Maj	Juni	Juli	Avg.	Sept.	Okt.	Nov.	Dec.	TOTAL
1960.	306.6	298.1	291.9	289.1	285.9	284.0	276.8	293.9	295.1	297.1	299.8	307.2	3535.4
1961.	310.0	306.7	304.0	309.2	310.2	315.2	314.2	314.3	300.0	306.0	320.8	301.5	3712.0
1962.	310.5	307.2	304.7	321.1	309.4	320.6	318.4	310.3	324.4	317.5	304.3	317.3	3765.8
1963.	319.3	321.7	342.4	317.4	347.8	338.3	345.6	340.0	346.0	350.7	344.4	348.7	4062.4
1964.	349.5	353.7	354.9	351.7	369.3	343.9	339.2	363.0	336.4	361.8	353.3	372.3	4248.9
1965.	368.3	371.4	376.3	403.2	370.2	407.9	421.0	407.3	416.8	390.6	415.3	421.3	4769.5
1966.	397.4	413.0	402.1	388.3	399.3	392.4	396.5	427.2	439.3	446.8	473.1	423.9	4999.4
1967.	444.4	448.0	434.6	438.6	438.7	440.7	430.1	393.8	407.9	394.3	367.1	377.8	5016.1
$\bar{x}$	350.8	352.5	351.4	353.6	353.9	355.4	355.2	356.2	358.2	358.1	359.8	358.8	4623.7
$\sigma$	46.4	51.3	47.4	47.5	47.5	49.5	51.6	46.5	52.2	47.9	55.0	45.0	557.4

Tabela 3.

Izravnate vrednosti izvoza Velike Britanije  
(pomoću metode G-1)

God.	Jan.	Feb.	Mart	Apr.	Maj	Juni	Juli	Avg.	Sept.	Okt.	Nov.	Dec.	TOTAL
1961.	329.1	295.9	315.2	306.5	310.6	311.0	306.0	281.8	264.1	292.7	317.2	316.3	3646.4
1962.	315.4	297.3	325.9	313.6	333.7	293.4	332.2	302.1	282.9	538.4	302.3	309.9	3747.2
1963.	327.1	322.4	369.8	331.7	364.8	342.6	325.3	324.5	308.1	336.1	339.0	341.3	4032.6
1964.	329.4	360.4	400.3	355.5	382.9	359.2	359.5	339.4	322.7	348.6	397.9	384.0	4340.1
1965.	391.2	393.7	397.3	399.6	434.1	417.1	407.9	360.5	348.6	404.8	446.7	426.7	4828.3
1966.	384.9	385.7	451.7	412.4	445.2	399.2	395.9	408.0	376.9	410.4	450.9	446.2	4967.5
1967.	400.8	438.4	436.9	437.0	416.4	410.3	416.3	395.4	392.4	435.1	415.9	432.4	5027.4
$\bar{x}$	354.0	356.3	365.3	365.2	384.0	361.8	363.3	344.5	328.0	366.6	381.4	379.5	
$\sigma$	33.7	49.7	48.0	47.6	47.2	45.4	40.7	43.1	43.9	47.2	57.0	53.2	

<sup>3</sup> Ekonomski analiza

Tabela 4.

## Izvoz Velike Britanije

1960—1967

(u milionima funti sterlinga)

God.	I kvartal	II kvartal	III kvartal	IV kvartal	TOTAL
1960.	920.1	899.6	814.5	902.6	3536.8
1961.	936.2	933.9	874.3	937.1	3681.5
1962.	931.9	982.2	902.8	976.6	3793.5
1963.	997.7	1031.9	974.2	1072.6	4076.4
1964.	1080.4	1090.7	975.7	1108.8	4255.6
1965.	1121.7	1200.8	1142.2	1254.5	4719.2
1966.	1266.7	1198.6	1206.0	1372.7	5044.0
1967.	1333.0	1334.2	1178.6	1162.5	5008.3
$\bar{x}$	1073.5	1084.0	1008.5	1098.4	
$\sigma$	148.0	141.0	139.2	151.5	

Tabela 5.

## Izravnate vrednosti izvoza Velike Britanije

(pomoću metode X-11)

God.	I kvartal	II kvartal	III kvartal	IV kvartal	TOTAL
1960.	899.9	878.3	865.1	891.6	3534.9
1961.	916.4	911.5	928.6	924.5	3681.1
1962.	914.3	958.1	958.4	961.5	3792.3
1963.	982.1	1006.1	1033.8	1053.2	4075.1
1964.	1067.0	1063.7	1034.0	1086.8	4251.5
1965.	1110.1	1171.7	1209.5	1228.1	4719.4
1966.	1222.1	1232.9	1275.4	1343.6	5073.9
1967.	1321.8	1303.3	1245.6	1196.0	5066.7
$\bar{x}$	1054.2	1065.7	1068.8	1085.7	
$\sigma$	146.1	145.6	145.6	149.6	

Tabela 6.

Izravnate vrednosti izvoza Velike Britanije  
(pomoću metode G-2)

God.	I kvartal	II kvartal	III kvartal	IV kvartal	TOTAL
1961.	924.8	887.4	965.3	912.0	3689.3
1962.	947.5	938.6	970.0	942.3	3798.3
1963.	1013.9	1042.4	1016.4	1007.9	4080.6
1964.	1018.6	1146.9	1031.8	1060.2	4257.5
1965.	1162.1	1193.8	1204.4	1167.7	4728.0
1966.	1190.7	1410.2	1185.5	1256.9	5043.3
1967.	1111.4	1446.0	1178.7	1270.4	5006.5
$\bar{x}$	1052.7	1152.1	1078.9	1088.2	
$\sigma$	96.1	200.9	98.6	135.0	

Prognozirane relativne vrednosti u narednim godinama

1968.	0.2817	0.1935	0.2845	0.2437
1969.	0.2239	0.2624	0.2451	0.2678
1970.	0.2363	0.2895	0.2255	0.2472

Tabela 7.

Izravnate vrednosti izvoza Velike Britanije  
(pomoću metode G-1)

God.	I kvartal	II kvartal	III kvartal	IV kvartal	TOTAL
1961.	936.2	955.8	870.9	932.6	3695.4
1962.	933.7	982.1	887.1	1000.8	3803.6
1963.	981.3	1012.1	972.5	1068.9	4034.7
1964.	1042.7	1068.2	1012.0	1113.9	4236.8
1965.	1191.8	1189.3	1137.2	1148.8	4667.1
1966.	1235.7	1238.4	1206.9	1332.2	5013.4
1967.	1303.3	1328.0	1167.1	1292.1	5090.5
$\bar{x}$	1089.2	1110.6	1036.2	1127.0	
$\sigma$	141.0	131.9	125.6	134.8	

Prognozirane relativne vrednosti u narednoj godini

1968.	0.2620	0.2622	0.2301	0.2456
-------	--------	--------	--------	--------

**PROJECTION PROGRAMME  
SOURCE STATEMENT**

**IBFTC V-D PR LIST, REF**

**THE V-D ADJUSTMENT PROGRAMME**

Written in Fortran IV, March 1969, by Vincentiu Dumitru for  
IBM 7044 at the International Computing Centre, United  
Nations — New York.

THE PROGRAMME PROVIDES THE ADJUSTMENT AND PROJECTION OF DATA  
AS PER THE ATTACHED REQUEST PAPERS OF MR. B. IVANOVICI,  
CHIEF, U. N. TRADE SECTION, USING TWO METHODS.  
IT CONSISTS OF A MAIN PROGRAMME PLUS A I.B.M. MATRIX INVERSE SUBROUTINE  
\*MINV\*IBM APPLICATION PRORGAMME, H20-0205-0, SYSTEM/360 SCIENTIFIC  
SUBROUTINE PACKAGE.

**SELECTION OF OPTIONS AND DATA FORMAT.**

**A) CONTROL SERIES CARDS.**

- DATA CARDS FOR EACH SERIES ARE PRECEDED BY THIS CARD.
  - COL. 1. \*1\* PUNCH.
  - COLS. 2-7. SERIES IDENTIFICATION CODE. MUST BE IDENTICAL TO  
COLS. 75-80 IN THE SERIES DATA CARDS. (NUMERIC AND/OR ALPHABETIC).
  - COLS. 8-67 SERIES TITLE. APPEARS AT THE OF PRINTED OUTPUT.
- OF SERIES.
- COLS. 71-73.
  - IF\*1\*IN COL. 71 AND\*0\*IN COL. 72, ONLY METHOD ONE WILL BE APPLIED.
  - IF\*0\*IN COL. 71 AND\*2\*IN COL. 72, ONLY METHOD TWO WILL BE APPLIED.
  - IF\*1\*IN COL. 71 AND\*2\*IN COL. 72, BOTH METHODS WILL BE APPLIED.
  - IF,1,IN COL. 73, THE INTERMEDIATE COMPUTATIONS WILL NOT BE PRINTED.
  - COLS. 74-75. NUMBER OF DATA CARDS IN SERIES (SEE COL. 80).
  - COLS. 76-77. NUMBER OF DATA FIELDS ON EACH CARD.
  - COL. 78. MAY BE\*1\* OR SPACE, IF \*1\*, ONLY THE INITIAL MATRIX  
AND FINAL COMPUTED MATRIX WILL BE PRINTED.
  - COL. 79-80. THE NUMBER OF YEARS FOR WHICH THE SERIES IS TO BE  
EXTENDED (NUMERIC). THE SUM OF COL. 80 AND COLS. 74-75 MUST BE NOT  
GREATER THAN 100.

**B) DATA CARDS.**

- COLS. 73-74. LAST TWO DIGITS OF YEAR.
- COLS. 75-80 SERIES IDENTIFICATION CODE.  
DATA CARDS IMMEDIATELY FOLLOW THE TITLE CARD.  
EACH CARD MAY CONTAIN UP TO ONE CALENDAR YEAR OF DATA.  
DATA FOR MONTH IS PUNCHED IN A 6-DIGIT FIELD,  
WITH IMPLIED DECIMAL POINTER AFTER THE 4TH DIGIT.

**C) A CARD PUNCHED\*7\*IN COL. 1 MUST FOLLOW THE FINAL DATA CARD.**

DOUBLE PRECISION AMAT, D, A3  
DIMENSION DATIN (24, 100), AMAT (24, 24), BMAT (24, 24), CMAT (24, 24)  
DIMENSION CTL (16), DTC (24), FMT (20), FMT2 (20), ADTL (100), WDAT (24, 100)  
DIMENSION LI (24), MI (24), FMT3 (20), NYR (100), A3 (3, 3), B3 (3), C3 (3)

**THE V-D ADJUSTMENT PROGRAMME**

```

5    DIMENSION FMT4 (25), ASTR (24, 24), AUNI (24, 24), FMT
6    5 (25), IXE (3)
7    DIMENSION WDT2 (24, 10C)
8    DATA Z/6HZ, /T/6HT /,TOT/6H TOTAL/, FORI/6H (1H,
9    DATA FORL/6HF9.2) /,FOR/6HF9.2, /,FORX/6H4X,
10   /,FORT/6HF 10.2,/
11   DATA FORI2/6H(1H, /,FORX2/6H10X, /,A/6HA6, /,FOR4/
12   /,6HF9.4,/
13   DATA FOR3L/6HF10.4) /,AN/6H19 /,AYR/6HYEAR /,FOR4L/
14   /,6HF9.4,/
15   DATA FIN/6H12, /,FAN/6HA2, /,FX/6H2X, /,FORX3/6H5X,/
16   DATA AS/6H** /,AVG/6H MEAN /,SGM/6H SIGMA,/FOR3/
17   /,6HF10.4,/
18   DATA FOR5/6HF 10.2, /,FOR5L/6HF10.2)/
19   NL = 0
20   GO TO 800
21
22   READ (5, 11) (CTL (J), J = 1, 12) MT1, MT2, NINT, NL, NC,
23   NP, IP
24   FORMAT (A1, A6, 10A6, 3X, I1, I1, I1, I2, I2, I1, I2)
25   IF (CTL (1). EQ. Z) GO TO 7000
26   IF (CTL(1). EQ. T) GO TO 15
27   GO TO 10
28
29   15 I = 0
30   FMT (1) = FORI
31   FMT (2) = FAN
32   FMT (3) = FIN
33   FMT (4) = FX
34   FMT (5) = FORT
35   FMT (6) = FX
36   FMT (7) = FAN
37   NT = NC - 1
38   DO 1 K = 1, NT
39
40   1 FMT (K + 7) = FOR
41   FMT (NC + 7) = FORL
42   FMT 2 (1) = FORI2
43   FMT2 (2) = FORX2
44   FMT2 (3) = A
45   FMT2 (4) = FORX
46   DO 2 K = 1, NT
47
48   2 FMT2 (K + 4) = FOR
49   FMT2 (NC + 4) = FORL
50   FMT3 (1) = FORI2
51   FMT5 (1) = FORI2
52   DO 3 K = 1, NT
53   FMT5 (K + 1) = FOR5
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110

```

111 4 FMT4 (K + 7) = FOR4  
 113 FMT4 (NC + 7) = FOR4L  
 114 20 I = I + 1  
 115 READ (5, 21) (DATIN (J, I), J = 1, 12), NOY, AID  
 123 21 FORMAT (12F6.2, I2, A6)  
 124 IF (I, GE. 2) GO TO 25  
 127 NOYP = NOY  
 130 NYR (1) = NOY  
 131 LYS = NOYP + NL - 1  
 132 25 IF (AID, NE, CTL (2)) GO TO 5000  
 135 IF ((NOYP + I - 1). NE. NOY) GO TO 5000  
 140 NYR (I) = NOY  
 141 IF (NOY, LT, LYS) GO TO 20  
 144 WRITE (2, 100)  
 145 WRITE (6, 100)  
 146 100 FORMAT (1H1, 41X, 49HUNITED NATIONS — E. S. A. —  
 STATISTICAL OFFICE, NEW YORK 1K/1H, 50X, 30H  
 INTERNATIONAL COMPUTING CENTRE/1H, 53X 24HV-D  
 ADJUSTMENT PROGRAMME)  
 147 WRITE (2, 105) (CTL (J), J = 3, 12)  
 154 WRITE (6, 105) (CTL (J), J = 3, 12)  
 161 105 FORMAT (1HO, 33X, 11A6)  
 162 WRITE (2, 110) AID  
 163 WRITE (6, 110) AID  
 164 110 FORMAT (1HO, 14HINITIAL MATRIX, 46X, 6HSERIES,  
 2X, A6)  
 165 WRITE (2, 115) AYR, TOT  
 166 WRITE (6, 115) AYR, TOT  
 167 115 FORMAT (1HO, A4, 6X, A6)  
 170 DTL = 0.0  
 171 DO 140 I = 1, NL  
 172 DO 120 J = 1, NC  
 173 DTL = DTL + DATIN (J, I)  
 174 ADTL (I) = 0.0  
 175 120 DTC (J) = DTC (J) + DATIN (J, I)  
 177 ADTL (I) = DTL  
 200 WRITE (2, FMT) AN, NYR (I), DTL, AS (DATIN (K, I),  
 K = NC)  
 205 WRITE (6, FMT) AN, NYR (I), DTL, AS, (DATIN (K, I),  
 K = 1, NC)  
 212 DTL = 0.  
 213 140 CONTINUE  
 215 WRITE (2, 222)  
 216 WRITE (6, 222)  
 217 WRITE (2, FMT2) TOT, (DTC (J), J = 1, NC)  
 224 WRITE (6, FMT2) TOT, (DTC (J), J = 1, NC)  
 231 FNL = NL  
 232 DO 150 J = 1, NC  
 233 150 DTC (J) = DTC (J)/FNL  
 235 WRITE (2, FMT2) AVG, (DTC (J), J = 1, NC)  
 242 WRITE (6, FMT2) AVG, (DTC (J), J = 1, NC)  
 247 DO 200 I = 1, NL  
 250 DO 200 J = 1, NC  
 251 WDAT (J, I) = DATIN (J, I)/ADTL (I)  
 252 200 CONTINUE

255 DO 201 I = 1, NL  
 256 DO 201 J = 1, NC  
 257 DATIN (J, I) = DATIN (J, I) — DTC (J)  
 260 201 DATIN (J, I) = DATIN (J, I)\*DATIN (J, I)  
 263 DO 203 J = 1, NC  
 264 203 DTC (J) = O. O  
 266 DO 205 I = 1, NL  
 267 DO 205 J = 1, NC  
 270 205 DTC (J) = DTC (J) + DATIN (J, I)  
 273 DO 207 J = 1, NC  
 274 207 DTC (J) = SQRT (DTC (J), FNL)  
 276 WRITE (2, FMT2) SGM, (DTC (J), J = 1, NC)  
 303 WRITE (6, FMT2) SGM, (DTC (J), J = 1, NC)  
 310 IF (NP, EQ, 1) GO TO 240  
 313 WRITE (2, 210) AID  
 314 210 FORMAT (1H1, 15HRELATIVE MATRIX, 45X, 6HSERIES,  
 2X, A6)  
 315 WRITE (2, 222)  
 316 WRITE (6, 222)  
 317 222 FORMAT (1HO)  
 320 DO 240 I = 1, NL  
 321 WRITE (2, FMT 3) (WDAT (J, I), J = 1, NC)  
 326 DO 240 J = 1, NC  
 327 WDT2 (J, I) = WDAT (J, I)  
 330 240 CONTINUE  
 333 STAB = WDAT (1, 2)  
 334 DO 245 I = 1, 24  
 335 DTC (I) = 0.0  
 336 DO 245 J = 1, 24  
 337 AMAT (J, I) = 0.0  
 340 ASTR (J, I) = 0.0  
 341 AUNI (J, I) = 0.0  
 342 BMAT (J, I) = 0.0  
 343 CMAT (J, I) = 0.0  
 344 245 CONTINUE  
 347 NTL = NL - 1  
 350 DO 250 L = 1, NC  
 351 DO 250 M = 1, NC  
 352 DO 250 K = 1, NTL  
 353 ASTR (M, L) = ASTR (M, L) + (WDAT (M, K)\*WDAT  
 (L, K))  
 354 BMAT (M, L) = BMAT (M, L) + (WDAT (L, K)\*WDAT  
 (M, K + 1))  
 355 250 CONTINUE  
 361 M2 = 0  
 362 IF ((MT1, EQ, 0), AND, (MT2, EQ, 2)) GO TO 1000  
 365 WRITE (6, 253) MTI  
 366 WRITE (2, 253) MTI  
 METHOD I  
 365 WRITE (6, 253) MTI  
 366 WRITE (2, 253) MTI

```

367 253 FORMAT (1H1, 118X, 6HMETHOD, 1X, I2)
370   BMULT = 1.0
371   EPS = 1.0
372   DO 255 L = 1, NC
373     DO 255 M = 1, NC
374       IF (ABS (ASTR (M, L) ). GE. EPS) GO TO 255
375       IF (ASTR (M, L). EQ. 0.0) GO TO 255
377         EPS = (ASTR (M, L) )
402
403   255 CONTINUE
404   FACT = 1.0
405   DO 260 K = 1,8
410     IF (EPS. GE. 1.0) GO TO 260
413     EPS = EPS*10.0
414     FACT = FACT*10.0
415   260 CONTINUE
417   270 CONTINUE
420     AMULT = FACT*BMULT
421     DO 300 L = 1, NC
422     DO 300 M = 1, NC
423     AMAT (M, L) = ASTR (M, L)*AMULT
424
427   300 CONTINUE
428     IF (NP. EQ. 1) GO TO 350
432     IF (BMULT. GT. 1.0) GO TO 350
435     WRITE (2, 310) AMULT, AID
436   310 FORMAT (1HO, 22HA-MATRIX MULTIPLIED BY, 1X, F10. 1,
437     27X, 6HSERIES, 2X, A6)
438     WRITE (2, 222)
440     DO 320 I = 1, NC
441   320 WRITE (2, FMT3) (AMAT (J, I), J = 1, NC)
442     WRITE (2, 330) AID
443     WRITE (2, 222)
451   330 FORMAT (1H1, 8HB-MATRIX, 52X, 6HSERIES 2X, A6)
452     DO 340 I = 1, NC
453   340 WRITE (2, FMT3) (BMAT (J, I), J = 1, NC)
454
461   350 CONTINUE
462     CALL MINV (AMAT, NC, 24, D, LI, MI, MIC, MFL)
466     IF (DABS (D). GT. 0.0) GO TO 370
471   360 WRITE (2, 365) AID
472     WRITE (6, 365) AID
473   365 FORMAT (1HO, 24HTHE A-MATRIX OF SERIES, A6, 2X,
474     11HIS SINGULAR)
475     BMULT = BMULT*10.0
476     IF (MIC. EQ. 0) GO TO 367
477     IF (AMULT. LE. 10000.0) GO TO 270
480
483   367 CONTINUE
484     WRITE (2, 368)
485
486   368 FORMAT (1HO, 49HTOO MANY TRAPS IN SUBROUTINE
487     MINV FOR THIS SERIES)

```

```

506   506 GO TO 800
507   370 CONTINUE
510   510 DO 371 I = 1,24
511   511 DO 371 J = 1,24
512   512 371 AMAT (J, I) = AMAT (J, I)*AMULT
515   515 WRITE (2, 373) AID
516   516 373 FORMAT (1HO, 9HINVERSE-A, 57X, 6HSERIES,
517     2X, A6)
517   517 WRITE (2, 222)
520   520 DO 375 I = 1, NC
521   521 375 WRITE (2, FMT5) (AMAT (J, I), J = 1,NC)
522   522 WRITE (2, 376) AID
527
530   530 376 FORMAT (1H1, 20HIDENTITY MATRIX AUNI, 46X, 6HSERIES,
531     2X, A6)
531   531 DO 377 I = 1, NC
532   532 DO 377 J = 1, NC
533   533 DO 377 K = 1, NC
534   534 AUNI (J, I) = AUNI (JI) + ASTR (K, I)*AMAT (J, K)
535   535 377 CONTINUE
541   541 WRITE (2, 222)
542   542 DO 378 I = 1, NC
543   543 378 WRITE (2, FMT3) (AUNI (J, I), J = 1, NC)
551   551 DO 380 L = 1, NC
552   552 DO 380 M = 1, NC
553   553 CMAT (M, L) = 0.0
554   554 DO 380 K = 1, NC
555   555 CMAT (M, L) = CMAT (M, L) + BMAT (L, K)*AMAT
556   556 (K, M)
556
562   562 380 CONTINUE
565   565 IF (NP. EQ. 1) GO TO 400
566   566 WRITE (2, 390) AID
567
570   570 390 FORMAT (1H1, 8HC-MATRIX, 52X 6HSERIES, 2X, A6)
571   571 WRITE (2, 222)
572   572 DO 395 I = 1, NC
573   573 395 WRITE (2, FMT3) (CMAT (J, I), J = 1, NC)
577
600   600 400 CONTINUE
601   601 DO 420 L = 1, NL
602   602 DO 420 M = 1, NC
603   603 DATIN (M, L) = 0.0
606
607   607 420 CONTINUE
608   608 DO 450 L = 1, NL
609   609 DO 450 M = 1, NC
610   610 DO 450 K = 1, NC
611   611 DATIN (M, L + 1) = DATIN (M, L + 1) + WDAT (K,
612   612 L)*CMAT (M, K)
616
621   621 450 CONTINUE
622   622 IF (IP. LT. 2) GO TO 465
623   623 I1 = NL + 1
624   624 I2 = NL + 1P - 1
625   625 DO 460 L = I1, I2
626   626 DO 460 M = 1, NC
627   627 DO 460 K = 1, NC
628
632   632 460 DATIN (M, L + 1) = DATIN (M, L + 1) + DATIN (K, L)*
633   633 CMAT (M, K)
634   634 I3 = NL + IP
635
636   636 3000 CONTINUE

```

```

634      WRITE (2, 470) AID
635      470 FORMAT (1H1,           24HCOMPUTED RELATIVE MATRIX,
               36X, 6HSERIES, 2X, A6)
636      WRITE (2, 222)
637      DO 500 I = 2, NL
640      WRITE (2, FMT3)  DATIN (J, I), J = 1, NC
645      500 CONTINUE
647      DO 550 I = 1, NL
650      DO 550 J = 1, NC
651      WDAT (J, I) = DATIN (J, I)*ADTL (I)
652      550 CONTINUE
655      WRITE (2, 222)
656      WRITE (6, 222)
657      WRITE (2, 570) AID
660      WRITE (6, 575) AID
661      570 FORMAT (1H1,           23HFFINAL ADJUSTED MATRIX, 37X,
               6HSERIES, 2 X, A6)
662      575 FORMAT (1H0,           23HFFINAL ADJUSTED MATRIX, 37X,
               6HSERIES, 2 X, A6)
663      WRITE (6, 580) AYR, TOT
664      WRITE (2, 580) AYR, TOT
665      580 FORMAT (1HO, A4, 6X, A6/)
666      DTL = 0.0
667      DO 590 I = 2, NL
668      DO 585 J = 1, NC
669      DTL = DTL + WDAT (J, I)
670      585 DTC (J) = DTC (J) + WDAT (J, I)
671      WRITE (2, FMT) AN, NYR (I), DTL, AS, (WDAT (K, I),
672                           K = 1, NC)
673      WRITE (6, FMT) AN, NYR (I), DTL, AS, (WDAT (K, I),
674                           K = 1, NC)
675      DTL = 0.0
676      590 CONTINUE
677      WRITE (2, 222)
678      WRITE (6, 222)
679      WRITE (2, FMI2) TOT, (DTC (J), J = 1, NC)
680      WRITE (6, FMIT2) TOT, (DTC (J), J = 1, NC)
681      FNL = NL - 1
682      DO 650 J = 1, NC
683      650 DTC (J) = DTC (J)/FNL
684      WRITE (2, FMT2) AVG, (DTC (J), J = 1, NC)
685      WRITE (6, FMT2) AVG, (DTC (J), J = 1, NC)
686      DO 701 I = 2, NL
687      DO 701 J = 1, NC
688      WDAT (J, I) = WDAT (J, I) - DTC (J)
689      701 WDAT (J, I) = WDAT (J, I)*WDAT (J, I)
690      DO 703 J = 1, NC
691      703 DTC (J) = 0.0
692      DO 705 I = 2, NL
693      DO 705 J = 1, NC

```

```

756      705 DTC (J) = DTC (J) + WDAT (J, I),
757      761 DO 707 J = 1, NC
758      762 707 DTC (J) = SQRT (DTC (J)/FNL)
759      764 WRITE (2, FMT2) SGM, (DTC (J), J = 1, NC)
760      771 WRITE (6, FMT2) SGM, DTC (J) J = 1, NC
761      776 WRITE (2, 750) AID
762      777 WRITE (6, 750) AID
763      1000 750 FORMAT (1H1,           23HRELATIVE PROJECTED DATA, 37X,
764                           6HSERIES, 2X, A6)
765      1001 WRITE (2, 115) AYR, TOT
766      1002 WRITE (6, 115) AYR, TOT
767      1003 DTL = 0.0
768      1004 DO 770 I = I1, I3
769      1005 NPY = NOYP + I - 1
770      1006 DO 760 J = 1, NC
771      1007 DTL = DTL + DATIN (J, I)
772      1011 DTLF = 1.0*DTL
773      1012 IF (M2, EQ. 1) DATIN (NC, I) = DTLF
774      1015 IF (M2, EQ.1) DTL = 1.0
775      1020 WRITE (2, FMT4) AN, NPY,     DTL, AS (DATIN (K, I),
776                           K = 1, NC)
777      1025 WRITE (6, FMT4) AN, NPY,     DTL, AS, (DATIN (K, I),
778                           K = 1, NC)
779      1032 DTL = 0.0
780      1033 770 CONTINUE
781      1035 800 CONTINUE
782      1036 DO 900 = 1, 100
783      1037 DO 900 J = 1,24
784      1040 DATIN (J, I) = 0.0
785      1041 WDAT (J, I) = 0.0
786      1042 900 CONTINUE
787      1045 DO 907 I = 1,24
788      1046 DO 907 J = 1,24
789      1047 CMAT (J, I) = 0.0
790      1050 907 CONTINUE
791      1053 K = LP-1
792      1054 910 DTC (J) = 0.0
793      1056 IF (NL, EQ. 0) GO TO 920
794      1061 IF (M2, EQ. 1) GO TO 920
795      1064 IF (MT2, EQ. 2) GO TO 1007
796      1067 920 CONTINUE
797      1070 DO 930 I = 1,100
798      1071 DO 930 J = 1,24
799      1072 WDAT (J, I) = 0.0
800      1073 WDT2 (J, I) = 0.0
801      1074 930 CONTINUE
802      1077 GO TO 10
803      1100 5000 WRITE (6, 5001) AID
804      1101 5001 FORMAT (1HO, 6HSERIES, 2X, A6, 2X, 17HIS A MIXED
805                           SERIES)
806      1102 GO TO 800
807      1103 7000 REWIND 2
808      1104 STOP

```

## METHOD II

```

1105 1000 CONTINUE
1106      WRITE (2, 1001) AID
1107 1001 FORMAT (1H1,     8HA-MATRIX, 52X, 6HSERIES, 2X, A6)
1108      WRITE (2, 222)
1109      DO 1002 I = 1, NC
1110
1111 1002 WRITE (2, FMT3) (ASTR (J, I), J = 1, NC)
1112      WRITE (2, 222)
1113      WRITE (2, 1003) AID
1114
1115 1003 FORMAT (1H1, 8HB-MATRIX, 52X, 6HSERIES, 2X, A6)
1116      DO 1005 I = 1, NC
1117
1118 1005 WRITE (2, FMT3) (BMAT (J, I), J = 1, NC)
1119
1120 1007 CONTINUE
1121      DO 1008 I = 1,100
1122      DO 1008 J = 1,24
1123      WDAT (J, I) = WDT2 (J, I)
1124
1125 1008 CONTINUE
1126      WRITE (2, 1009) MT2
1127      WRITE (6, 1009) MT2
1128
1129 1009 FORMAT (1H1, 118X, 6HMETI(O, 1X, J2)
1130      DO 1010 L = 1,100
1131      DO 1010 M = 1,24
1132      DATIN (M, L) = 0.0
1133
1134 1010 CONTINUE
1135      DO 2000 LP = 1, NC
1136      K = LP-1
1137      L = LP + 1
1138      DO 1015 I = 1,3
1139      B3 (I) = 0.0
1140      DO 1015 J = 1,3
1141
1142 1015 A3 (J, I) = 0.0
1143      DO 1150 I = 1,3
1144      M = K + I
1145      IF (M, GT, NC) M = M - NC
1146      IF (L, GT, NC) L = L - NC
1147      B3 (I) = BMAT (L, M)
1148      IXE (I) = M
1149      DO 1150 J = 1,3
1150      N = K + J
1151      IF (N, GT, NC) N = N - NC
1152
1153 1150 A3 (J, I) = ASTR (N, M)
1154      IF (NINT, EQ, 1) GO TO 1240
1155
1156 1170 WRITE (2, 1200)
1157
1158 1200 FORMAT (1H1, 50X, 25HINTERMEDIATE COMPUTATIONS//)
1159
1160 1210 DO 1240 I = 1,3
1161      WRITE (2, 1230) (A3 (J, I), J = 1,3), B3 (I)
1162
1163 1230 FORMAT (1H, 40X, 3F8. 4, 10X, F8. 4)
1164
1165 1240 CONTINUE
1166      CALL MINV (A3, 3, 3, D, LI, MI, MIC, MFL)
1167      IF (D, EQ, 0.0, OR, MIC, EQ, 1, OR, MFL, EQ, 1, OR,
1168      MFL, EQ, 3) M2 = 1
1169
1170      DO 1250 I = 1,3
1171      C3 (I) = 0.0
1172      DO 1250 J = 1,3
1173
1174 1250 C3 (I) = C3 (1) + A3 (J, I)*B3 (J)
1175      IF (NINT, EQ, 1) GO TO 1281

```

```

1243      WRITE (2, 1153)
1244 1153 FORMAT (1HO)
1245      DO 1270 I = 1,3
1246      WRITE (2, 1280) (A3 (J, I), J = 1,3), C3 (I)
1247
1248 1280 FORMAT (1H, 57X, 3F9. 1, 10X, F8. 4)
1249
1250 1281 CONTINUE
1251      IE 1 = I XE (1)
1252      IE 2 = I XE (2)
1253      IE 3 = I XE (3)
1254      IF (LP, EQ, NC) GO TO 1283
1255      CMAT (LP + 1, IE1) = C3 (1)
1256      CMAT (LP + 1, IE2) = C3 (2)
1257      CMAT (LP + 1, IE3) = C3 (3)
1258      GO TO 1285
1259
1260 1283 CONTINUE
1261      CMAT (1, IE1) = C3 (1)
1262      CMAT (1, IE2) = C3 (2)
1263      CMAT (1, IE3) = C3 (3)
1264
1265 1285 CONTINUE
1266      IF (NINT, EQ, 1) GO TO 1291
1267      WRITE (2, 1290) IE1, IE2, IE3
1268
1269 1290 FORMAT (1HO, 3 (2X, I4) )
1270
1271 1291 CONTINUE
1272      IF (LP, EQ, (NC - 1) ) GO TO 1400
1273      IF (LP, EQ, NC) GO TO 1450
1274      DO 1300 I = 1, NL
1275
1276 1300 DATIN (IE2, I + 1) = C3 (1)*WDAT (IE1, I) + C3 (2)*
1277      WDAT (IE2, I) + C3 (3)*WDAT (IE3,
1278      1300 CONTINUE
1279      GO TO 1490
1280
1281 1400 CONTINUE
1282      NZ = NL - 1
1283      DO 1430 J = 1, NZ
1284      J1 = J
1285      J2 = J
1286      J3 = J + 1
1287
1288 1430 DATIN (IE2, J + 1) = C3 (1)*WDAT (IE1, J1) + C3 (2)*
1289      WDAT (IE2, J2) + C3 (3)*WDAT (IE3, J3)
1290      DATIN (IE2, NL + 1) = 0.0
1291      GO TO 1490
1292
1293 1450 CONTINUE
1294      DATIN (IE2, 2) = STAB
1295      DO 1470 J = 2, NL
1296      J1 = J-1
1297      J2 = J
1298      J3 = J
1299
1300 1470 DATIN (IE2, J + 1) = C3 (1)*WDAT (IE1, J1) + C3 (2)*
1301      WDAT (IE2, J2) + C3 (3)*WDAT (IE3, J3)
1302
1303 1490 CONTINUE
1304      IF (NINT, EQ, 1) GO TO 2000
1305      WRITE (2, 222)
1306      WRITE (2, 1153)
1307      DO 1500 I = 1, NL
1308
1309 1500 WRITE (2, FMT3) (DATIN (J, I) J = 1, NC)
1310
1311 2000 CONTINUE
1312      WRITE (2, 222)
1313      WRITE (2, 2500) AID

```

```

1361 2500 FORMAT (1HO, 20HC-MARTIX, TRANSPOSED., 40X, 6HSE-
      RIES, 2X, A6)
1362 WRITE (2, 222)
1363 DO 2700 I =1, NC
1364 2700 WRITE (2, FMT3) (CMAT (J, I), J = 1, NC)
1372     I1 = NL + 1
1373     I2 = NL
1374     I3 = NL + 1
1375     M2 = 1
1376 GO TO 3000
1377 4000 CONTINUE
1400 WRITE (2, 4005) AID
1401 WRITE (6, 4005) AID
1402 4005 FORMAT (1HO, 24HA SUBMATRIX OF A-SERIES, A6, 2X,
      11HIS SINGULAR)
1403 GO TO 800
1404 END

```

(Rad primljen juna 1969.)

#### ADJUSTMENT, INTERPOLATION AND EXTRAPOLATION OF SEASONAL TIME SERIES

by Branislav IVANOVIC

##### *Summary*

If in the course of  $N$  years one measures characteristic  $X$  each year  $n$  times and if  $x_{ij}$  is its  $j$ -th value in the  $i$ -th year, the matrix  $x_{ij}$  represents a time series. The variations of values in one year and the values corresponding to different years can result from random factors, seasonal influences and the general tendencies of movements of the examined phenomenon. If the series is sufficiently long a cyclical characteristic can manifest itself.

If the seasonal influences are very pronounced and the tendency of interseasonal variations more or less stable the relative adjusted values of the year  $(t+1)$  are given in (1.1), the coefficients  $k_{ij}$  are elements of the law of movement (1.2) calculated by the use of (1.3), where the elements of matrix  $A_{ii}$  are given by (1.4') and (1.4'').

Forecasts of future series are given in (1.5) while (1.6) enables us to reconstruct earlier series.

If the interseasonal variation is insignificant in relation to random variation of the phenomenon there is no further question of seasonal influences on the law of movement. The new law of movement then is given by (1.9) and the adjusted series in (1.10).

On the basis of (1.11) we can forecast future series and with the aid of (1.2) reconstruct earlier series.

We use exports of Great Britain in the period 1960—1967 for application and comparison of this method.

The monthly data for exports are give in Table (1) while Table (4) presents the quarterly values. Tables (2) and (5) represent adjusted values obtained by method X-11, Tables (3) and (7) by method G-1, and Table (6) by method G-2.

Finally, let us note that the Fortran programme drawn up by Mr. Vincentu Dumitru, of the UN Electronic Centre, for two alternatives of the proposed method contains no more than ten pages.