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ENTERPRISE PRICING OF MULTIPHASE PRODUKTION BY USING INPUT-OUTPUT MODELLING

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ABSTRACT

The paper presents and discusses some theoretical and practical aspects of enterprise input/output modelling. The emphasis is given to a cost-oriented price model enabling industrial marketing management to improve decision making associated with pricing policy. The usefulness of the discussed models is illustrated by a hypothetical, but very indicative numerical example where the effects on cost pricing for two production alternatives were analysed.

1. INTRODUCTION

The economic function of the industrial enterprise is to supply a product that will satisfy market demand and sell at a profit. It is the production function that satisfies this demand by producing the product in the required quantity, of the right quality, at the required time, and at a competitive cost. To satisfy the last requirement it is necessary to implement such marketing activities which ensure us a large enough share of the market to provide the volume necessary to enable the products to be made at competitive costs. It is characteristic for some industrial enterprises that fixed costs represent an important share of production costs. The purpose is to develop a suitable price model and to illustrate by numerical examples the impact of different production level on product pricing. Considering a production of technologically dependent products, it is obvious that adequate cost analysis is no longer possible by using breakeven analysis [2, 6] but only on the basis of enterprise input/output modelling.

Enterprise models discussed by Müller—Mehrbach [4], Pichler [5], Teusch—Schlütter [7], and Meško [3] depended on the characteristics of the production process and for operative purposes represent

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only a specific form of the input/output modelling in the sense of Leontief [1]. The assumptions used in these models can sometimes limit their applicability. For this reason, a suitable enterprise input/output model is developed where the problems arise from the two basic assumptions: the first is that each production centre produces only one kind of output (product), and the second is that for a production of one kind of output the same technology is used; those are successfully solved and tested on the hypothetical numerical example where the effects on cost pricing for two production alternatives were analysed.

2. METHODOLOGY

2.1. *General Approach*

A general approach to enterprise input/output modelling begins with structural analysis of the production process which has to be decomposed to such functional entireties (production centres) whose outputs can be measured in the economical and technical sense of the word. Natural flows which describe the relation between the production centres in the reproduction process as well as the relation between the production centres and markets can be mathematically expressed in matrix forms as

$$x = Ax + q \tag{2.1}$$

where $x \in \mathbb{R}^n$ whose components represent the volume of the outputs measured in quantity units (q.u.), A is a n x n matrix whose elements define the direct consumption of the intermediate outputs in reproduction process, and $q \in \mathbb{R}^n$ whose components indicate sales volumes. The above form of input/output model can be used to solve some practical problems. Thus, on the basis of the relation

$$q = c - Ac \tag{2.2}$$

where $c \in \mathbb{R}^n$ whose components represent production capacities which are supposed to be independent, it is possible to determine for each capacity utilization alternative the maximum volumes of outputs destined for a sale. The solution of (2.1) for x

$$x = (E - A)^{-1} q \tag{2.3}$$

is possible to use for analysing the impact of the different sales volume alternatives on the level of production. Here the matrix $(E - A)^{-1}$ represents inversion of the matrix $(E - A)$ where E is a n x n diagonal matrix whose elements are all unity. It is important to distinguish the meaning of the element a_{ij} of the matrix A from the element r_{ij} of the matrix $(E - A)^{-1}$. While the element a_{ij} defines the volume of direct consumption of the i-th output in the production of the j-th output unity, so the element r_{ij} defines the total volume consumption (direct and indirect) of the i-th output caused by the production of the j-th output unity which is destined for a sale.

The volume of costs inputs, x^d, for a given sales alternative is defined by the relation

$$x^d = A^d x \tag{2.4}$$

where $x^d \in \mathbb{R}^m$ whose components denote the volume of inputs, and A^d is a m x n matrix whose elements represent the coefficients which define direct consumption of the inputs. The matrix A^d and vector x^d can be partitioned to distinguish, for instance, inputs of raw materials from direct labour inputs. The relations (2.1)—(2.4) are the fundamental ones which enable one to analyse and plan material flows in the production process, and at the same time they represent the base for other enterprise input/output models.

2.2. *Two Extensions of the Basic Input/Output Models*

As already mentioned, the applicability of enterprise input/output modelling can be seriously endangered if some of the outputs can be produced by different technological methods or if more than one output in the same production centres can be produced which could be the case in the production of by-products.

The first of these two problems can be satisfactorily solved by introducing a fictitious centre and by adding new ones for each product which can be produced by different technological methods. Thus, it is possible to analyse the impact of the natural (physical) and/or financial flows of this production centre on other production centres. Relations between the production centres representing different technological methods and the relating fictitious production centre are expressed by input coefficients, the values of which denote the share of each technology in the production of 1q.u. of fictitious product and are obtained by experience or by optimization. All other input coefficients of the considered fictitious production centre have a value equal to zero.

The second problem which arises if the same production centre produces more outputs can be solved by introducing the matrix A^+ whose elements a^+_{ij} denote the quantity of the i-th by-product (P^+_i) which is obtained by the production of the j-th output unity. In the prices models, which will be discussed in the next section, the by-products are treated as final products which should strengthen the competitive position of the main products. For this reason, coefficients of the matrix A^+ in the price models appear with a negative sign.

2.3. *Input/Output Price Model*

For the purpose of developing a suitable input/output price model, total costs are defined as direct costs (costs of the original costs inputs), derivative costs (costs due to the consumption of intermediate products), and added costs (here fixed costs with the required profit) which can be expressed in matrix form as

$$X = \hat{x}(-A^{+T}p^{+} + A^{dT}p^{d}) + B^{T}X + d \tag{2.5}$$

where $X \in \mathbb{R}^n$ whose components denote the amount of total costs, $\hat{x}$ is a $n \times n$ diagonal matrix whose elements are equal to volume outputs, $p^{+} \in \mathbb{R}^r$ whose components designate the selling prices of the products obtained in the production process, $p^{d} \in \mathbb{R}^m$ whose components denote the purchasing prices of the costs inputs, B is a $n \times n$ matrix whose elements define the consumption shares of the i-th output in the production of the j-th output unity, and $d \in \mathbb{R}^n$ whose components represent the amount of the fixed costs with the required profit. It is assumed that the distribution of total costs and fixed costs of each production centre on other production centres is defined by consumption share of the j-th production centre in the production of the i-th centre. A letter "T" denotes transpose of the matrix. Solution (2.5) for the vector X has the form

$$X = (E - B^{T})^{-1} \; (-\hat{x}A^{+T}p^{+} + \hat{x}A^{dT}p^{d}) + (E - B^{T})^{-1} d \tag{2.6}$$

A distribution matrix B is possible to express in the form

$$B = \hat{x}^{-1} \cdot A \cdot \hat{x}$$

from which follows

$$B^{T} = \hat{x} \cdot A^{T} \cdot \hat{x}^{-1}$$

Considering the substitution for the matrix B and price equation $p^{c} = \hat{x}^{-1} \cdot X$, the relation (2.6) can be rewritten in the form

$$p^{c} = \hat{x}^{-1}X = \hat{x}^{-1}\hat{x}(-A^{+T}p^{+} + A^{dT}p^{d}) + \hat{x}^{-1}\hat{x}A^{T}\hat{x}^{-1}\hat{x}p^{c} + \hat{x}^{-1}d \tag{2.7}$$

from which follows the full cost price model

$$p^{c} = (E - A^{T})^{-1} (-A^{+T}p^{d} + A^{dT}p^{d}) + (E - A^{T})^{-1}\hat{x}^{-1}d \tag{2.8}$$

or, after substitution of expression $(E - A^{T})^{-1} (-A^{+T}p^{+} + A^{dT}p^{d})$ by vector p^{v}

$$p^{c} = p^{v} + (E - A^{T})^{-1}\hat{x}^{-1}d \tag{2.9}$$

which can be used as a main analytical tool for analysing the effects of the changing operating production level and changed prices of costs inputs, technological and institutional conditions on full cost pricing. For sales management, however, it is necessary to know, besides full cost prices, the selling prices covering only all variable costs in a product unity destined for a sale. In the price model (2.9) the components of vector $p^v \in \mathbb{R}^n$ represent variable costs' prices which indicate the lower bound under which is, from the economical point of view, no more reasonable to produce and sell the products.

3. NUMERICAL EXAMPLE

To illustrate the practical value of the presented input/output models which will be demonstrated on two production alternatives, suppose that an enterprise produces three kind of outputs: the first one can be produced by two technologies. At the production of the second product as well as at the production of the third product one by-product is obtained. In the production process it is necessary to use two kinds of raw materials, and the direct labour which is decomposed on the three degree of the labour requirement (degree of skill). Production capacities which are independent and fixed costs with the required profit for each production centre are given. Natural data are measured in quantitative (q.u.) respectively in labour units (l.u.), costs and prices, however, are expressed in USA-dollars. The data of an enterprise, represented by the production system extended on five production centres, are given in the matrix form:

$$A = \begin{bmatrix} 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.125 & 0 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 0 & 0 & 0 & 0.025 & 0 \\ 0 & 0 & 0 & 0 & 0.02 \end{bmatrix} \quad p^+ = \begin{bmatrix} 8 \\ 25 \end{bmatrix}$$

$$A^m = \begin{bmatrix} 0.4 & 0.5 & 0 & 5 & 3.2 \\ 3 & 5 & 0 & 7.1 & 6.5 \end{bmatrix} \quad p^m = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}$$

$$A^l = \begin{bmatrix} 3 & 7 & 0 & 18 & 15 \\ 1 & 3 & 0 & 12 & 6 \\ 1 & 1 & 0 & 2 & 1 \end{bmatrix} \quad p^l = \begin{bmatrix} 0.01 \\ 0.05 \\ 0.15 \end{bmatrix}$$

$$d^T = [2{,}000{,}000 \quad 1{,}000{,}000 \quad 0 \quad 4{,}000{,}000 \quad 3{,}000{,}000]$$

$$c^T = [6{,}880{,}000 \quad 10{,}320{,}000 \quad 17{,}200{,}000 \quad 6{,}000{,}000 \quad 4{,}800{,}000]$$

Some of the example data need to be explained. The fourth and the fifth production centre appears as supplier ($a_{45} = 0.5$) and as consumer ($a_{54} = 0.125$) at the same time. The element $\overset{+}{a}_{14} = 0.025$

of matrix A^+ means that at producing q.u. of P_4 there are obtained 0.025q.u. of P_6. For the production of q.u. of P_5 there are needed, for instance 3.2q.u. of the first costs input $(\overset{m}{a}_{15} = 3.2)$ and, for instance 6 l.u. of the second degree of labour requirement $(\overset{1}{a}_{26} = 6)$.

To make the right pricing decisions provoked by different business situations, examine the possible cost-oriented selling prices' variations relating to 50% and to 80% operating production level. The results obtained on the basis of (2.2) and (2.8) are represented in Table 1 and Table 2. As expected, cost-oriented selling prices relating to 80% capacity utilization are about 7—9% lower compared with the prices obtained at a 50% operating production level which may suggest support for a pricing policy which also considers sales volumes.

Table 1. — *Cost-oriented selling prices calculated at 50% capacity utilization*

Production centre/product	Variable costs' prices	Full cost prices	Production quantity	Sales quantity
P_i	$\overset{v}{p}_i$	$\overset{c}{p}_i$	x_i	q_i
P_1	1.03	1.611	3,440,000	—
P_2	1.67	1.864	5,160,000	—
P_3	1.414	1.763	8,600,000	200,000
P_4	8.725	11.105	3,000,000	1,800,000
P_5	8.777	11.565	2,400,000	2,025,000

Table 2. — *Cost-oriented selling prices calculated at 80% capacity utilization*

Production centre/product	Variable costs' prices	Full cost prices	Production quantity	Sales quantity
P_i	$\overset{v}{p}_i$	$\overset{c}{p}_i$	x_i	q_i
P_1	1.03	1.393	5,504,000	—
P_2	1.67	1.791	8,256,000	—
P_3	1.414	1.632	13,760,000	320,000
P_4	8.725	10.212	4,800,000	2,880,000
P_5	8.777	10.520	3,840,000	3,240,000

4. CONCLUSION

The paper presents input/output price models which can be very useful for industrial marketing management as they enable the analysis of the effects of changing operating production level and changed purchasing prices, technological and institutional conditions on full cost pricing. Two theoretical problems were successfully solved, but among other problems there remains the question of model approach relating to semi-fixed costs which were not the subject of the price models discussed.

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OBLIKOVANJE PROIZVAJALČEVIH CEN V VEČFAZNI PROIZVODNJI NA PODLAGI MEDSEKTORSKEGA MODELIRANJA

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P o v z e t e k

V članku je obravnavan podjetniški medsektorski cenovni model, ki je primeren za oblikovanje proizvajalčevih cen v večfazni proizvodnji. Uporabnost modela, ki vključuje tudi dve teoretični izboljšavi, je prikazana na pirejenem numeričnem primeru, kjer sta analizirani dve proizvodni varianti glede na njun vpliv na višino proizvodnih cen.